Name:____________________________________.

MATH 115 - SEC 011, WINTER 2011. QUIZ 5 TIME LIMIT: 20 MINUTES

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Good luck!

Problem 1. Find the quadratic polynomial $g(x) = ax^2 + bx + c$ which best fits the function $f(x) = e^x$ at $x = 0$, in the sense that

$$g(0) = f(0), \text{ and } g'(0) = f'(0), \text{ and } g''(0) = f''(0).$$

Taking the first and second derivative, we get

$$g'(x) = 2ax + b, \; g''(x) = 2a, \; f'(x) = f''(x) = e^x$$

So

$$g(0) = c, g'(0) = b, g''(0) = 2a,$$
$$f(0) = f'(0) = f''(0) = 1.$$

Matching the equations

$$\begin{aligned} g(0) &= f(0) \\ g'(0) &= f'(0) \\ g''(0) &= f''(0), \end{aligned}$$

we get

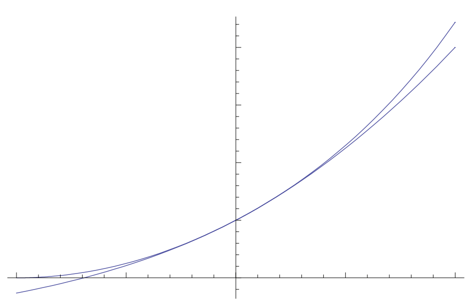
$$\begin{aligned} c &= 1 \\ b &= 1 \\ 2a &= 1 \end{aligned}$$

The quadratic polynomial which best fits the exponential function is

$$g(x) = 1 + x + \frac{x}{2}.$$

Using your calculator, sketch the graphs of f and g on the same axes. What do you notice?

In the window [-1,1], Figure shows that the exponential function and the polynomial computed above agree better than the tangent line.



Date: February 16, 2011.

Problem 2. The period, T, of a pendulum is given in terms of its length, ℓ, by

$$T = 2\pi\sqrt{\frac{\ell}{g}},$$

where g is the acceleration due to gravity (a constant).

- Find $\frac{dT}{d\ell}$. *Show your work. Specify what rules you are using.* You are **NOT** allowed to use rules that we haven't seen in class.

$$T = 2\pi\frac{1}{\sqrt{g}}e^{1/2}.$$

Since this is a power function, the derivative is

$$\frac{dT}{d\ell} = \frac{2\pi}{\sqrt{g}}\frac{1}{2\sqrt{\ell}} = \frac{\pi}{\sqrt{g\ell}}.$$

- What is the sign of $\frac{dT}{d\ell}$? What does it tell you about the period of the pendulum?

We notice that the sign is always positive. This implies that the period of the pendulum increases as the length increases.

Problem 3. For what value(s) of a are $y = a^x$ and $y = 1 + x$ tangent at $x = 0$? *Explain.*

The two functions e^x and $1+x$ are equal to 1 at $x = 0$. This means that the two graphs intersect at the y-axis. In order for the graphs to be tangent at $x = 0$, we need that the derivatives are the same at $x = 0$.

For $y = a^x$,

$$\frac{dy}{dx} = \frac{d}{dx}(a^x) = ln(a)a^x,$$

and for $y = 1 + x$,

$$\frac{d}{dx}(1 + x) = 1.$$

So, evaluating those expressions at $x = 0$ we get

$$ln(a) = 1,$$

so $a = e$.

Problem 4. State the *quotient* rule. **Be specific.**

If $u = f(x)$ and $v = g(x)$ are differentiable, then

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2},$$

or equivalently,

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}\cdot v - u\cdot\frac{dv}{dx}}{v^2}.$$

Problem 5. Find the derivative of the following functions.

(a) $f(x) = x \cdot e^x$

Using the product rule, and the derivatives of exponential functions, we get

$$f'(x) = e^x + xe^x = (x+1)e6x$$

(b) $f(x) = x \cdot 2^x$

$$f'(x) = 2^x + x\ln(2)2^x = 2^x(1 + \ln(2)x)$$

(c) $g(t) = \frac{t-4}{t+4}$

Using the quotient rule, we get

$$g'(t) = \frac{1\cdot(t+4) - (t-4)cdot1}{(t+4)^2} = \frac{t+4-t+4}{(t+4)^2}$$
$$= \frac{8}{(t+4)^2}$$

(d) $y = (t^2+3)\cdot e^t$

Here we use the product rule and the derivative of polynomials and exponential functions, and get

$$\frac{dy}{dx} = 2te^t + (t^3+3)e^t = e^t(t^2+2t+3)$$