Name:______________________________________.

MATH 115 - SEC 011, WINTER 2011. QUIZ 6
TIME LIMIT: 25 MINUTES

INSTRUCTOR: GERARDO HERNÁNDEZ

Good luck!

Problem 1. Differentiate the following functions. If you need more space, use the last page for your computations.

(a) $y = \sqrt{z}\, e^{-z}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{z}}e^{-z} + \sqrt{z}(-e^{-z})$$
$$= e^{z}\left(\frac{1-2z}{2\sqrt{z}}\right)$$

(b) $y = \left(\frac{x^2+2}{ln(x)}\right)^2$

$$\frac{dy}{dx} = 2\left(\frac{x^2+2}{\ln x}\right)\cdot\left(\frac{2x\ln(x) - (x^2+2)\frac{1}{x}}{\ln(x)^2}\right)$$
$$= \frac{2(x^2+2)\cdot(2x^2\ln(x) - x^2 - 2)}{x\ln(x)^3}$$

(c) $f(x) = 2x\ tan(cos(x))$

Using the chain rule twice, and the product rule we get

$$f'(x) = 2\tan(\cos(x)) + 2x\frac{1}{\cos^2(cos(x))}(-\sin(x))$$
$$= 2\tan(\cos(x)) - \frac{2x\sin(x)}{\cos^2(\cos(x))}$$

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(d) $r(\theta) = arctan(\theta)\ \sqrt{cos(3\theta)}$

Using the product rule, and chain rule twoce, we get

$$r'(\theta) = \frac{1}{1+\theta^2}\sqrt{\cos(3\theta)} + \arctan(\theta)\ \frac{1}{2\sqrt{\cos(3\theta)}}3\cdot(-\sin(3\theta))$$

$$= \frac{\sqrt{\cos(3\theta)}}{1+\theta^2} - \frac{3\arctan(\theta)\sin(3\theta)}{2\sqrt{\cos(3\theta)}}.$$

(e) $f(x) = e^{-2x}\ sin(x)$

Here we use the product rule and the derivative of exponential functions to obtain

$$f'(x) = -2e^{-2x}\sin(x) + e^{-2x}\cos(x)$$

$$= e^{-2x}\left(-2\sin(x) + \cos(x)\right)$$

(f) $G(x) = \frac{sin^2(x)-1}{cos^2(x)+1}$

Using the quotient rule, and derivatives of trigonometric functions, we find

$$G'(x) = \frac{2\sin(x)\cos(x)(\cos^2(x)+1) - (\sin^2(x)-1)(2\cos(x))(-\sin(x))}{(\cos^2(x)+1)^2}$$

$$= \frac{\sin(x)\cos(x)\left(2\cos^2(x)+2+2\sin^2(x)-2\right)}{\left(\cos^2(x)+1\right)^2}$$

$$= \frac{2\sin(x)\cos(x)}{\left(\cos^2(x)+1\right)^2}$$

(g) $g(t) = cos(ln(t))$

Chain rule applied several times gives:

$$g'(t) = -\sin(\ln(t)) \cdot \frac{1}{t} = -\frac{\sin(\ln(t))}{t}.$$

(h) $T(u) = arctan\left(\frac{u}{1+u}\right)$

Chain rule plus quotient rules gives:

$$T'(u) = \frac{1}{1+\left(\frac{u}{1+u}\right)^2}\left(\frac{1\cot(1+u) - 1\cdot u}{(1+u)^2}\right) = \frac{1}{1+\frac{u^2}{(1+u)^2}}\frac{1}{(1+u)^2}$$

$$= \frac{1}{(1+u)^2+u^2}$$

Problem 2.

- For $x > 0$, find and simplify the derivative of $f(x) = arctan(x) + arctan(1/x)$

  Using the derivative of arctan and the chain rule, we get for $x > 0$:

$$f'(x) = \frac{1}{1+x^2} + \frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-x^{-2}\right)$$

$$= \frac{1}{1+x^2} - \frac{1}{\left(1+\left(\frac{1}{x}\right)^2\right)x^2} = \frac{1}{1+x^2} - \frac{1}{1+x^2} = 0$$

- What does the result tell you about f?

  Since the derivative is zero, then the function is constant for $x > 0$. In fact, using properties of trigonometric functions, one can show that

$$\arctan(x) + \arctan\left(\frac{1}{x}\right) = \frac{\pi}{2}$$

  for $x > 0$.