0423.1
2012

Hw 11

Section 5.3 # 33

One solution of the differential equation is given. Find the general solution.

$y^{(3)} + 3y'' - 54y = 0$; $y = e^{3x}$

Answer: The characteristic equation is:

$r^3 + 3r^2 - 54 = 0$

Since e^{3x} is a solution $\Rightarrow r = 3$ is a zero of the char. polynomial

$\Rightarrow r^3 + 3r^2 - 54 = (r-3)(r^2 + br + c)$ for some constants b, c.

$\Rightarrow r^3 + 3r^2 - 54 = r^3 + br^2 + cr - 3r^2 - 3br - 3c$

$= r^3 + r^2(b-3) + r(c-3b) - 3c$

Collecting the corresponding coefficients we get:

$-3c = -54 \Rightarrow c = 18$

$c - 3b = 0 \Rightarrow c = 3b \Rightarrow b = 6$

$b - 3 = 3$ ✓

$\Rightarrow r^3 + 3r^2 - 54 = (r-3)(r^2 + 6r + 18)$

The other two solutions are $r = \frac{-6 \pm \sqrt{36 - 4 \times 1 \times 18}}{2} = -3 \pm 3i$

$\Rightarrow e^{-3x} \cos 3x$ $e^{-3x} \sin 3x$ are the other l.i. solution.

Therefore, the general solution is:

$y(x) = c_1 e^{3x} + e^{-3x}(c_2 \cos 3x + c_3 \sin 3x)$.

Section 5.4 #18

Most grand father clocks have pendulums with adjustable lengths. One such clock ~~uses~~ loses 10 min per day when the length of its pendulum is 30 in. With what length pendulum will this clock keep perfect time?

Answer: Using problem 5 (also included in the homework), we know that for two pendulums of lengths L_1, L_2 and - when located at the respective distances R_1 and R_2 from the center of the earth - with periods P_1, P_2,

$$\frac{P_1}{P_2} = \frac{R_1\sqrt{L_1}}{R_2\sqrt{L_2}}$$

Let's find P_1

for this clock, $L_1 = 30$ in

In 1 day it oscillates $24\times60^2 - 10\times60$ times

(~~because~~ assuming a perfect pendulum oscillates 1 time per second).

$$\Rightarrow P_1 = \frac{24\times60^2\ s}{24\times60^2 - 10\times60}$$

Assume P_2 ~~is the~~ corresponds to the perfect clock.

$\Rightarrow P_2 = 1\,s$. Find L_2 ($R_1 = R_2$ because we assume we are on the same location)

$$\Rightarrow L_2 = \frac{P_2^2}{P_1^2} L_1 = \frac{1\,s^2}{\left(\frac{24\times60^2\ s}{24\times60^2-10\times60}\right)^2} \times 30\text{ in} \approx 29.59\text{ in}$$

$\therefore$ The length the pendulum must have to keep a perfect time is $L_2 = 29.59$ in.