LO507.1
2012

Section 7.1 #16.

Find a general solution of the system

$x' = 8y$

$y' = -2x$

Answer:

Taking another derivative to the first eqn. we get:

$x'' = 8y' = 8 \cdot (-2x) = -16x$

$\Rightarrow x'' + 16x = 0$

Char poly: $r^2 + 16 = 0 \quad r = \pm 4i$

$\Rightarrow \cos(4t), \sin(4t)$ are two l.i. solutions

$x(t) = c_1 \cos(4t) + c_2 \sin(4t)$

$\Rightarrow y = \frac{1}{8} x' = \frac{1}{8}\left(-4c_1 \sin(4t) + 4c_2 \cos(4t)\right)$

$= -\frac{1}{2} c_1 \sin(4t) + \frac{1}{2} c_2 \cos(4t)$

$\therefore \; x = c_1 \cos(4t) + c_2 \sin(4t)$

$y = \frac{1}{2} c_2 \cos(4t) - \frac{1}{2} c_1 \sin(4t)$

Section 7.2 #16

Verify that the given vectors are solutions of the given system. Then use the Wronskian to show that they are linearly independent. Finally, write the general solution of the system:

$$\bar{X}' = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} \bar{X}, \quad \bar{X}_1 = e^{3t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \bar{X}_2 = e^{2t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Answer: $\bar{X}_1' = 3e^{3t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} \bar{X}_1 = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} e^{3t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = e^{3t}\begin{bmatrix} 4-1 \\ -2-1 \end{bmatrix} = e^{3t}\begin{bmatrix} 3 \\ -3 \end{bmatrix} = \bar{X}_1' \checkmark$$

$$\bar{X}_2 = e^{2t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} \qquad \bar{X}_2' = 2e^{2t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = e^{2t}\begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} \bar{X}_2 = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} e^{2t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} = e^{2t}\begin{bmatrix} 4-2 \\ -2-2 \end{bmatrix} = e^{2t}\begin{bmatrix} 2 \\ -4 \end{bmatrix} = \bar{X}_2' \checkmark$$

The Wronskian is given by

$$W(x) = \begin{vmatrix} e^{3t} & e^{2t} \\ -e^{3t} & -2e^{2t} \end{vmatrix} = -2e^{5t} + e^{5t} = -e^{5t} \neq 0$$

Therefore, $\bar{X}_1$ and $\bar{X}_2$ are linearly independent.

The general solution is then given by:

$$\bar{X}(t) = c_1\bar{X}_1(t) + c_2\bar{X}_2(t) = c_1 e^{3t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{2t}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} ce^{3t} + c_2 e^{2t} \\ -c_1 e^{3t} - 2c_2 e^{2t} \end{bmatrix}$$