4.1 # 26

Express the vector t as a linear combination of u, v, w

$t = (5, 30, -21)$, $u = (5, 2, -2)$, $v = (1, 5, -3)$, $w = (5, -3, 4)$.

Answer: Want to find a, b, c such that

$$t = au + bv + cw$$

$$\Rightarrow a \cdot \begin{pmatrix} 5 \\ 2 \\ -2 \end{pmatrix} + b \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} + c \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ -21 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} 5 & 1 & 5 \\ 2 & 5 & -3 \\ -2 & -3 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 30 \\ -21 \end{bmatrix}$$

$\Rightarrow$ Augmented coeff. matrix is:

$$\begin{bmatrix} 5 & 1 & 5 & 5 \\ 2 & 5 & -3 & 30 \\ -2 & -3 & 4 & -21 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} 5 & 1 & 5 & 5 \\ 2 & 5 & -3 & 30 \\ 0 & 2 & 1 & 9 \end{bmatrix} \xrightarrow[5 \times R_2]{2 \times R_1} \begin{bmatrix} 10 & 2 & 10 & 10 \\ 10 & 25 & -15 & 150 \\ 0 & 2 & 1 & 9 \end{bmatrix}$$

$$\xrightarrow[\frac{1}{2}R_1]{-R_1 + R_2} \begin{bmatrix} 5 & 1 & 5 & 5 \\ 0 & 23 & -25 & 140 \\ 0 & 2 & 1 & 9 \end{bmatrix} \xrightarrow{(-11)R_3 + R_2} \begin{bmatrix} 5 & 1 & 5 & 5 \\ 0 & 1 & -36 & 41 \\ 0 & 2 & 1 & 9 \end{bmatrix}$$

$$\xrightarrow{(-2)R_2 + R_3} \begin{bmatrix} 5 & 1 & 5 & 5 \\ 0 & 1 & -36 & 41 \\ 0 & 0 & 73 & -73 \end{bmatrix} \xrightarrow{\frac{1}{73}R_3} \begin{bmatrix} 5 & 1 & 5 & 5 \\ 0 & 1 & -36 & 41 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$x_3 = -1$, $x_2 - 36x_3 = 41 \Rightarrow x_2 = -36 + 41 = \cancel{\ \ } 5$

$5x_1 + x_2 + 5x_3 = 5 \Rightarrow 5x_1 = 5 - \cancel{41} - 5(-1) = + \cancel{\ \ } 5 \Rightarrow x_1 = +1$

$\cancel{a = -\frac{77}{5},\ b = -77 \ldots = 1} \Rightarrow a = +1,\ b = 5,\ c = -1$

$\therefore t = u + 5v - w$

Suppose that A is an $n \times n$ matrix and that k is a constant scalar. Show that the set of vectors x such that $Ax = kx$ is a subspace of $\mathbb{R}^n$.

$W = \{x \in \mathbb{R}^n \mid Ax = kx\}$

Answer:

Clearly $0 \in W \Rightarrow W \neq \emptyset$ $\quad W \subset \mathbb{R}^n$, $\mathbb{R}^n$ is a vector space.

We need to show two properties

(a) Let $u, v \in W$ be any two vectors in W.

$\Rightarrow Au = ku$, $Av = kv$

We want to show that $u + v \in W$

$A(u+v) = Au + Av = ku + kv = k(u+v)$

Therefore $u + v \in W$

(b) Let $u \in W$ and $c \in \mathbb{R}$ any scalar.

$\Rightarrow Au = ku$

We want to show that $cu \in W$

$A(cu) = cAu = cku = k(cu)$

Therefore $cu \in W$

So, W is not empty, and W is closed under addition and multiplication by scalars.

Therefore W is a subspace.