(0326.1)

2012

Section 4.5

Problem 3 Find both a basis for the row space and a basis for the column space of:

$$\begin{bmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \end{bmatrix}$$

Answer:

Row operations:

$$\xrightarrow[-R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & -4 & -3 & -7 \\ 0 & 7 & 7 & 21 \\ 0 & 6 & 6 & 18 \end{bmatrix} \xrightarrow[\frac{1}{6}R_3]{\frac{1}{7}R_2} \begin{bmatrix} 1 & -4 & -3 & -7 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow{-R_2+R_3} \begin{bmatrix} 1 & -4 & -3 & -7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{4R_2+R_1} \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = E$$

reduced echelon form

A basis for the row space is

$$\{[1 \ 0 \ 1 \ 5], [0 \ 1 \ 1 \ 3]\}$$

For the column space, we need to identify the pivot columns, which are 1 and 2.

A basis for the column space is;

$$\Rightarrow \text{A basis } \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} \right\}$$

Problem 17: Find a basis T for $\mathbb{R}^3$ that contains the vectors $v_1 = (1,2,2)$, and $v_2 = (2,3,3)$

Answer: Consider $\{v_1, v_2, e_1, e_2, e_3\}$

This is a linearly dependent set that spans $\mathbb{R}^3$

Use algorithm used in class to extract a basis from that list containing v_1, v_2.

Form the matrix A with those as its columns vectors

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 1 \end{bmatrix}$$ and proceed by row operations:

$$\xrightarrow[-2R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -2 & 1 & 0 \\ 0 & -1 & -2 & 0 & 1 \end{bmatrix} \xrightarrow{-R_1+R_2} \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -2 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

This is already in echelon form (not reduced), and the pivot colums are 1, 2, and 4

$\Rightarrow$ A basis T for $\mathbb{R}^3$ containing v_1, v_2 is

$T = \{v_1, v_2, e_2\}$.