

MATH 320 - SEC 001, SPRING 2012. PRACTICE EXAM

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1. For

$$\begin{cases} \frac{dy}{dx} = (e^{y+2} - 1) y (y - 2) \\ y(0) = y_0 \end{cases}$$

- (a) Sketch, roughly, a slope field and classify all the critical points.
- (b) Determine (from your sketch), the asymptotic behavior of the solution for $y_0 = -1$ as $t \rightarrow \infty$.

2. Solve

$$\begin{cases} (x + y)y' = x - y \\ y(1) = 0 \end{cases}$$

Give the range of validity of the solution

3. Consider the initial value problem

$$\begin{cases} \frac{dy}{dx} = -\frac{5}{2}x^4y^3 \\ y(0) = -1 \end{cases}$$

- (a) Find $y(x)$ explicitly. For what values of x is the solution defined?
- (b) Use one step of the modified Euler's method with step size h to find an approximation for $y(h)$

4 Write the following systems as $\mathbf{Ax} = \mathbf{b}$ and determine for what values of k the system has (i) a unique solution, (ii) no solution, and (iii) infinitely many solutions.

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 4 \\ 2x_1 + 3x_2 - x_3 &= k \\ -2x_1 + x_2 - 3x_3 &= 2 \end{aligned}$$