MATH 322 - SEC 001, SPRING 2013. HOMEWORK 10

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Due : Friday, April 26

Please show all your work and/or justify your answers for full credit.

Problem 1: Consider the Sturm-Liouville problem with boundary conditions of the third kind:

$$\begin{cases} \frac{d^2\phi}{dx^2} + \lambda\phi = 0 \\ h_1\phi(0) - \frac{d\phi}{dx}(0) = 0 \\ h_2\phi(L) + \frac{d\phi}{dx}(L) = 0 \end{cases}$$

Show that $\lambda = 0$ is an eigenvalue of the Sturm-Liouville problem if and only if the parameters h_1, h_2 satisfy the equation of the two-sheeted hyperbola

$$h_1 + h_2 + Lh_1h_2 = 0.$$

Problem 2: (*Textbook problem 5.4.1*) Consider

$$c\rho\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(K_0\frac{\partial u}{\partial x}\right) + \alpha u,$$

where c, ρ, K_0, α are functions of x, subject to

$$\begin{aligned} u(0,t) &= 0, \\ u(L,t) &= 0, \\ u(x,0) &= f(x). \end{aligned}$$

Assume that the appropriate eigenfunctions are known

(a) Show that the eigenvalues are positive if $\alpha < 0$.
(b) Solve the initial value problem
(c) Briefly discuss $\lim_{t\to\infty} u(x,t)$

Problem 3: (*Textbook problem 5.6.2*) Consider the eigenvalue problem

$$\frac{d^2\phi}{dx^2} + (\lambda - x^2)\phi = 0$$

subject to

$$\frac{d\phi}{dx}(0) = 0, \ \frac{d\phi}{dx}(1) = 0.$$

Show that $\lambda > 0$ (be sure to show that $\lambda \neq 0$).

Problem 4: (*Textbook problem 5.7.1*) Determine an upper and a (non-zero) lower bound for the lowest frequency of vibration of a non-uniform string fixed at $x = 0$ and $x = 1$ with $c^2 = 1 + 4\alpha^2(x - \frac{1}{2})^2$. **Show your work.**