MATH 322 - SEC 001, SPRING 2013. HOMEWORK 12

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Due : Friday, May 10

Please show all your work and/or justify your answers for full credit.

Problem 1: (*Textbook problem 5.9.3*) Consider for $\lambda >> 1$

$$\frac{d^2\phi}{dx^2} + [\lambda\sigma(x) + q(x)]\,\phi = 0.$$

(a) Substitue

$$\phi = A(x)\exp\left[i\lambda^{1/2}\int_0^x \sigma^{1/2}(z)dz\right].$$

Determine a differential equation for $A(x)$.

(b) Let $A(x) = A_0(x) + \lambda^{-1/2}A_1(x) + \ldots$. Solve for A_0 and $A_1(x)$. Verify equation (5.9.8).

(c) Suppose that $\phi(0) = 0$. Use $A_1(x)$ to improve (5.9.9).

(d) Use part (c) to improve (5.9.10) if $\phi(L) = 0$.

(d) Obtain a recursion formula for $A_n(x)$.

Problem 2: (*Textbook problem 6.2.4*) Suppose that we did not know equation (6.2.15) in the textbook, but thought it possible to approximate d^2f/dx^2 by an unknown linear combination of there functionns values, $f(x_0 - \Delta x), f(x_0), f(x_0 + \Delta x)$:

$$\frac{d^2 f}{dx^2} \approx af(x_0 - \Delta x) + bf(x_0) + cf(x_0 + \Delta x).$$

Determine a, b and c by expanding the right-hand side in a Taylor series around x_0 using (6.210) and (6.2.11) and equating coefficients through d^2f/dx^2.

Problem 3: (*Textbook problem 6.3.1*)

(a) Show that the truncation error for our numerical scheme, (6.3.3), becomes much smaller if $k\Delta t/(\Delta x)^2 = 1/6$. *Hint: u satisfies the partial differential equation in (6.3.1).*

(b) If $k\Delta t/(\Delta x)^2 = 1/6$, determine the order of magnitude of the truncation error.

Problem 3: (*Textbook problem 6.3.7*) Numerically compute solutions to the heat equation with the temperature initially given in Fig. 6.3.4. Use (6.3.16)-(6.3.18) with N=10. Do for various s (discuss stability):

$$s = 0.49,\ s = 0.50,\ s = 0.51,\ s = 0.52$$

Note: If you create your own code, include it in your homework. If you don't have access to matlab or any other compiler, do it by hand and report the numerical results in a table (3 time steps).