MATH 322 - SEC 001, SPRING 2013. HOMEWORK 9

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Due : Friday, April 19

Please show all your work and/or justify your answers for full credit.

Problem 1: (*Textbook problem 5.3.2*) Consider

$$\rho \frac{\partial^2 u}{\partial t^2} = T_0 \frac{\partial^2 u}{\partial x^2} + \alpha u + \beta \frac{\partial u}{\partial t}.$$

(a) Give a brief physical interpretation. What signs must α and β have to be physical?

(b) Allow ρ, α, β to be functions of x. Show that separation of variables works only if $\beta = c\rho$, where c is a constant.

(c) If $\beta = c\rho$, show that the spatial equation is a Sturm-Liouville differential equation. Solve the time equation.

Problem 2: (*Textbook problem 5.3.3*) Consider the non-Sturm-Liouville differential equation

$$\frac{d^2\phi}{dx^2} + \alpha(x)\frac{d\phi}{dx} + \left[\lambda\beta(x) + \gamma(x)\right]\phi = 0.$$

Multiply this equation by $H(x)$. Determine $H(x)$ such that the equation may be reduced to the standard Sturm-Liouville form

$$\frac{d}{dx}\left[p(x)\frac{d\phi}{dx}\right] + \left[\lambda\sigma(x) + q(x)\right]\phi = 0.$$

Given $\alpha(x), \beta(x)$ and $\gamma(x)$, what are $p(x), \sigma(x)$, and $q(x)$?

Problem 3: (*Textbook problem 5.3.6*) For the Sturm-Lioville eigenvalue problem

$$\frac{d^2\phi}{dx^2} + \lambda\phi = 0, \text{ with } \frac{d\phi}{dx}(0) = 0 \text{ and } \phi(L) = 0,$$

verify the following general properties:

(a) There is an infinite number of eigenvalues with a smallest but no largest

(b) The nth eigenfunction has $n-1$ zeros

(c) The eigenfunctions are complete and orthogonal

(d) What does the Rayleigh quotient say concerning negative and zero eigenvalues?

Problem 4: (*Textbook problem 5.3.7*) Which of the statements 1-5 of the theorems of this section are valid for the following eigenvalue problem?

$$\frac{d^2\phi}{dx^2} + \lambda\phi = 0$$

with

$$\phi(-L) = \phi(L)$$
$$\frac{d\phi}{dx}(-L) = \frac{d\phi}{dx}(L).$$

Problem 5: (*Textbook problem 5.3.8*) Show that $\lambda \geq 0$ for the eigenvalue problem

$$\frac{d^2\phi}{dx^2} + (\lambda - x^2)\phi = 0 \text{ with } \frac{d\phi}{dx}(0) = 0, \frac{d\phi}{dx}(1) = 0.$$

Is $\lambda = 0$ an eigenvalue?