MATH 322 , SPRING 2013. PRACTICE PROBLEMS FOR MIDTERM 2

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Problem 1 Compute the Fourier series of the indicated functions

(a) $f(x) = \cos^3 x$, $-\pi \le x \le \pi$.

(b) $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 \le x < \pi \end{cases}$

Problem 2 Sketch the Fourier cosine series of $f(x) = \sin(\pi x/L)$. Briefly discuss.

Problem 3 If $f(x) = \begin{cases} x^2, & x < 0 \\ e^{-x}, & x > 0 \end{cases}$, what are the even and odd parts of $f(x)$?

Problem 4 Fourier series can be defined on other intervals besides $-L \le x \le L$. Suppose that $g(y)$ is defined for $a \le y \le b$. Represent $g(y)$ using periodic trigonometric functions with period $b - a$. Determine formulas for the coefficients. *Hint:* Use the linear transformation

$$y = \frac{a+b}{2} + \frac{b-a}{2L}x.$$

Problem 5 Using the textbook formula (3.3.13), determine the Fourier cosine series of $\sin \pi x/L$.

Problem 6 Show that the derivative of an even function is an odd function.

Problem 7 Show that the derivative of an odd function is an even function.

Problem 8 Determine whether or not the indicated function is piecewise smooth.

(a) $f(x) = |x|^{3/2}$, $-2 < x < 2$

(b) $f(x) = [x] - x$, $0 < x < 3$ ($[x]$ = integer part of x)

(c) $f(x) = x^4 \sin(1/x)$, $-1 < x < 1$

(d) $f(x) = e^{-1/x^2}$, $-1 < x < 1$

Problem 9 Consider the heat equation with a known source $q(x,t)$:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + q(x,t) \text{ with } u(0,t) = 0 \text{ and } u(L,t) = 0.$$

Assume that $q(x,t)$ (for each $t > 0$) is a piecewise function of x. Also assume that u and $\partial u/\partial x$ are continuous functions of x (for $t > 0$) and $\partial^2 u/\partial x^2$ and $\partial u/\partial t$ are piecewise smooth. Thus,

$$u(x,t) = \sum_{n=1}^{\infty} b_n(t) \sin\frac{n\pi x}{L}.$$

What ordinary differential equation does $b_n(t)$ satisfy? Do not solve this differential equation.

Problem 10 Solve the following non homogeneous problem

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + e^{-t} + e^{-2t}\cos\frac{3\pi x}{L} \text{ [assume that } 2 \ne k(3\pi/L)^2]$$

subject to

$$\frac{\partial u}{\partial t}(0,t) = 0, \ \frac{\partial u}{\partial x}(L,t) = 0, \ \text{and } u(x,0) = f(x).$$

Use the following method. Look for the solution as a Fourier cosine series. Justify all differentiations of infinite series (assume appropriate continuity).

Problem 11 Let $f(x)$, $0 \leq x < L$, satisfy $f(x) = f(L-x)$. Let u be the solution of the wave equation with initial conditions $u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$. Show that $u(x, L/2c) = 0$ for $0 < x < L$.

Problem 12

(a) Using the textbook formulas (3.3.11) and (3.3.12), obtain the Fourier cosine series of x^2

(b) From part (a), determine the Fourier sine series of x^3

(c) Generalize part (b) in order to derive the Fourier sine series of x^m, m odd.