Erratum to
"Semicompleteness of homogeneous quadratic vector fields"

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In our article *Semicompleteness of homogeneous quadratic vector fields* (Ann. Inst. Fourier, Grenoble **56**, 5 (2006) 1583–1615) the proof of Proposition 3.2 is incomplete. We hereby complete the proof, fully establishing this proposition as well as Theorem A.

To prove the proposition it suffices to exhibit a vector field such that any deformation preserving the eigenvalues of its radial orbits comes from a linear change of coordinates. We claim that this happens for the vector field

$$X = z_1(z_1 - 7z_2 + 5z_3)\frac{\partial}{\partial z_1} + z_2(z_2 - 7z_3 + 5z_1)\frac{\partial}{\partial z_2} + z_3(z_3 - 7z_1 + 5z_2)\frac{\partial}{\partial z_3}.$$

This vector field is isochoric and has seven distinct non-degenerate radial orbits. We will consider the family of vector fields $f : \mathbf{C}^3 \to V_3$ given by

$$(\beta_1, \beta_2, \beta_3) \to X + \left(\sum_{j=1}^{3} \beta_j z_j\right) \sum_{i=1}^{3} z_i \frac{\partial}{\partial z_i}.$$

Notice that when $(\beta_1, \beta_2, \beta_3) = (-1, -1, -1)$ we obtain a multiple of the vector field X_0 appearing on page 1603 of the cited article. The arguments given there guarantee that *any deformation of the foliation induced in* $\mathbf{CP}^2$ *by the vector field* X_0 *that preserves its Baum-Bott indexes is necessarily given by a linear change of coordinates*. This implies that if X admits a deformation preserving its eigenvalues then this deformation belongs, up to a linear change of coordinates, to the family f. Let $\tau_4(\beta)$, $\tau_1(\beta)$ and $\tau_2(\beta)$ denote, respectively, the sum of the eigenvalues of the radial orbits $\rho_4 = [1 : 0 : 0]$, $\rho_1 = [0 : 1 : 0]$ and $\rho_2 = [0 : 0 : 1]$ of the vector field $f(\beta_1, \beta_2, \beta_3)$. An explicit calculation shows that

$$\left(\frac{1}{\tau_4} + \frac{1}{4}, \frac{1}{\tau_1} + \frac{1}{4}, \frac{1}{\tau_2} + \frac{1}{4}\right) = \left(-\frac{1}{4}\beta_1, -\frac{1}{4}\beta_2, -\frac{1}{4}\beta_3\right).$$

In this way, no isospectral deformations are to be found within the family f and, in consequence, X does not admit an isospectral deformation. This completes the proof of Proposition 3.2.

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