Examen II

Resolver los siguientes límites.

(1) $\lim\limits_{x\to-\infty} \frac{\sqrt{x^2+3x+1}}{\sqrt{16x^2+x+2}}$

Solución

$$\begin{aligned}
\lim_{x\to-\infty} \frac{\sqrt{x^2+3x+1}}{\sqrt{16x^2+x+2}} &= \sqrt{\lim_{x\to-\infty} \frac{x^2+3x+1}{16x^2+x+2}} \\
&= \sqrt{\lim_{x\to-\infty} \frac{x^2\left(1+\frac{3}{x}+\frac{1}{x^2}\right)}{x^2\left(16+\frac{1}{x}+\frac{2}{x^2}\right)}} \\
&= \sqrt{\lim_{x\to-\infty} \frac{1+\frac{3}{x}+\frac{1}{x^2}}{16+\frac{1}{x}+\frac{2}{x^2}}} \\
&= \sqrt{\frac{1}{16}} \\
&= \frac{1}{4}
\end{aligned}$$

(2) $\lim\limits_{x\to\infty} \sqrt{x^2+100x}-x$

Solución

$$\begin{aligned}
\lim_{x\to\infty} \sqrt{x^2+100x}-x &= \lim_{x\to\infty} \left(\sqrt{x^2+100x}-x\right)\left(\frac{\sqrt{x^2+100x}+x}{\sqrt{x^2+100x}+x}\right) \\
&= \lim_{x\to\infty} \frac{(x^2+100x)-x^2}{\sqrt{x^2+100x}+x} \\
&= \lim_{x\to\infty} \frac{100x}{\sqrt{x^2(1+\frac{100}{x})}+x} \\
&= \lim_{x\to\infty} \frac{100x}{x\sqrt{1+\frac{100}{x}}+x} \\
&= \lim_{x\to\infty} \frac{100x}{x(\sqrt{1+\frac{100}{x}}+1)} \\
&= \lim_{x\to\infty} \frac{100}{\sqrt{1+\frac{100}{x}}+1} \\
&= \frac{100}{\sqrt{1}+1} \\
&= \frac{100}{2} \\
&= 50
\end{aligned}$$

(3) $\lim_{x\to -1^-} \frac{1}{(x+1)^2}$

Solución

El límite diverge a $+\infty$ ya que si

$$x \to -1^-,$$

entonces

$$(x+1)^2 \to 0^+$$

(4) $\lim_{x\to\infty} \sqrt{x^2-3x+1} - \sqrt{3x^2-2}$

Solución

$$\begin{aligned}
&\lim_{x\to\infty} \sqrt{x^2-3x+1} - \sqrt{3x^2-2} \\
&= \lim_{x\to\infty} \left(\sqrt{x^2-3x+1} - \sqrt{3x^2-2}\right)\left(\frac{\sqrt{x^2-3x+1}+\sqrt{3x^2-2}}{\sqrt{x^2-3x+1}+\sqrt{3x^2-2}}\right) \\
&= \lim_{x\to\infty} \frac{(x^2-3x+1)-(3x^2-2)}{\sqrt{x^2-3x+1}+\sqrt{3x^2-2}} \\
&= \lim_{x\to\infty} \frac{-2x^2-3x+3}{\sqrt{x^2(1-\frac{3}{x}+\frac{1}{x^2})}+\sqrt{x^2(3-\frac{2}{x^2})}} \\
&= \lim_{x\to\infty} \frac{x(-2x-3+\frac{3}{x})}{x\sqrt{1-\frac{3}{x}+\frac{1}{x^2}}+x\sqrt{3-\frac{2}{x^2}}} \\
&= \lim_{x\to\infty} \frac{x(-2x-3+\frac{3}{x})}{x\left(\sqrt{1-\frac{3}{x}+\frac{1}{x^2}}+\sqrt{3-\frac{2}{x^2}}\right)} \\
&= \lim_{x\to\infty} \frac{-2x-3+\frac{3}{x}}{\sqrt{1-\frac{3}{x}+\frac{1}{x^2}}+\sqrt{3-\frac{2}{x^2}}}
\end{aligned}$$

Si $x\to\infty$, entonces $-2x\to-\infty$, lo que implica que el numerador de la última expresión diverge a $-\infty$. El denominador congerve a $1+\sqrt{3}$. Por lo tanto el límite que se busca diverge a $-\infty$.

(5) $\lim_{x\to\infty} \frac{\sqrt[3]{x}}{\sqrt[3]{2x+\sqrt{x}}}$

Solución

$$\begin{aligned}
\lim_{x\to\infty} \frac{\sqrt[3]{x}}{\sqrt[3]{2x+\sqrt{x}}} &= \sqrt[3]{\lim_{x\to\infty} \frac{x}{2x+\sqrt{x}}} \\
&= \sqrt[3]{\lim_{x\to\infty} \frac{x}{x(2+\frac{1}{\sqrt{x}})}} \\
&= \sqrt[3]{\lim_{x\to\infty} \frac{1}{2+\frac{1}{\sqrt{x}}}} \\
&= \sqrt[3]{\frac{1}{2}}
\end{aligned}$$

(6) $\lim\limits_{x\to 1} \frac{x^2}{1-x^2}$

Solución El límite no existe ya que

$$\lim_{x\to 1^-} \frac{x^2}{1-x^2}$$

diverge a $+\infty$ y

$$\lim_{x\to 1^+} \frac{x^2}{1-x^2}$$

diverge a $-\infty$.

(7) $\lim\limits_{x\to 0^+} x\sqrt{1+\frac{1}{x^2}}$

Solución

$$\begin{aligned} \lim_{x\to 0^+} x\sqrt{1+\tfrac{1}{x^2}} &= \lim_{x\to 0^+} \sqrt{x^2\left(1+\tfrac{1}{x^2}\right)} \\ &= \lim_{x\to 0^+} \sqrt{x^2+1} \\ &= \sqrt{1} \\ &= 1 \end{aligned}$$

(8) $\lim\limits_{x\to 0} \frac{\sin 5x}{\sin 7x}$

Solución

$$\begin{aligned} \lim_{x\to 0} \tfrac{\sin 5x}{\sin 7x} &= \lim_{x\to 0} \tfrac{(5x)(7x)\sin 5x}{(5x)(7x)\sin 7x} \\ &= \lim_{x\to 0} \tfrac{5}{7}\tfrac{\sin 5x}{5x}\tfrac{7x}{\sin 7x} \\ &= \tfrac{5}{7} \end{aligned}$$

(9) $\lim\limits_{x\to 0} \frac{\sin^2 x}{x\tan x}$

Solución

$$\begin{aligned} \lim_{x\to 0} \tfrac{\sin^2 x}{x\tan x} &= \lim_{x\to 0} \tfrac{\sin x\sin x\cos x}{x\sin x} \\ &= \lim_{x\to 0} \tfrac{\sin x\cos x}{x} \\ &= \lim_{x\to 0} \tfrac{\sin x}{x}\cos x \\ &= \cos 0 \\ &= 1 \end{aligned}$$

4 (10) $\lim_{x\to 0} \frac{\sin x \tan x}{x^2}$

Solución

$$\begin{aligned}
\lim_{x\to 0} \frac{\sin x \tan x}{x^2} &= \lim_{x\to 0} \frac{\sin x \sin x}{(x)(x)\cos x} \\
&= \lim_{x\to 0} \frac{\sin x}{x}\frac{\sin x}{x}\frac{1}{\cos x} \\
&= \frac{1}{\cos 0} \\
&= 1
\end{aligned}$$