LP Cycling Example

This example has the virtue of suffering from no roundoff errors when run on a computer. Cycling in LP's remains rare and so many implementations do not implement an anti-cycling rule. We use Anstee's pivot rules (which are otherwise known as the standard rules) to pivot into the basis the variable with the largest coefficient in the z row (and in the case of ties take the variable of smallest index) and for the leaving variable we break ties, if necessary, by choosing the variable of smallest index. We typically expect you to follow these pivot rules on test questions.

LP Cycling Example

dictionary 1

$$\begin{array}{rclrrrr} x_5 & = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\ x_6 & = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\ x_7 & = & 1 & -x_1 & & & \\ z & = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 \end{array}$$

LP Cycling Example

dictionary 1

$$\begin{array}{rcrrrrr} x_5 & = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\ x_6 & = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\ x_7 & = & 1 & -x_1 & & & \\ z & = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 \end{array}$$

x_1 enters

LP Cycling Example

dictionary 1

$$\begin{array}{rcrrrrr} x_5 & = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\ x_6 & = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\ x_7 & = & 1 & -x_1 & & & \\ z & = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 \end{array}$$

x_1 enters and x_5 leaves

LP Cycling Example

dictionary 1

$$\begin{array}{rcrrrrr} x_5 & = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\ x_6 & = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\ x_7 & = & 1 & -x_1 & & & \\ z & = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 \end{array}$$

x_1 enters and x_5 leaves

dictionary 2

$$\begin{array}{rcrrrrr} x_1 & = & & -2x_5 & +11x_2 & +5x_3 & -18x_4 \\ x_6 & = & & x_5 & -4x_2 & -2x_3 & +8x_4 \\ x_7 & = & 1 & +2x_5 & -11x_2 & -5x_3 & +18x_4 \\ z & = & & -20x_5 & +53x_2 & +41x_3 & -204x_4 \end{array}$$

LP Cycling Example

dictionary 2

$$
\begin{array}{rcrrrrr}
x_1 & = & & -2x_5 & +11x_2 & +5x_3 & -18x_4 \\
x_6 & = & & x_5 & -4x_2 & -2x_3 & +8x_4 \\
x_7 & = & 1 & +2x_5 & -11x_2 & -5x_3 & +18x_4 \\
z & = & & -20x_5 & +53x_2 & +41x_3 & -204x_4
\end{array}
$$

LP Cycling Example

dictionary 2

$$
\begin{array}{rclrrrr}
x_1 & = & & -2x_5 & +11x_2 & +5x_3 & -18x_4 \\
x_6 & = & & x_5 & -4x_2 & -2x_3 & +8x_4 \\
x_7 & = & 1 & +2x_5 & -11x_2 & -5x_3 & +18x_4 \\
z & = & & -20x_5 & +53x_2 & +41x_3 & -204x_4
\end{array}
$$

x_2 enters

LP Cycling Example

dictionary 2

$$
\begin{array}{rclrrrr}
x_1 & = & & -2x_5 & +11x_2 & +5x_3 & -18x_4 \\
x_6 & = & & x_5 & -4x_2 & -2x_3 & +8x_4 \\
x_7 & = & 1 & +2x_5 & -11x_2 & -5x_3 & +18x_4 \\
z & = & & -20x_5 & +53x_2 & +41x_3 & -204x_4
\end{array}
$$

x_2 enters and x_6 leaves

LP Cycling Example

dictionary 2

$$
\begin{array}{rcrrrrr}
x_1 & = & & -2x_5 & +11x_2 & +5x_3 & -18x_4 \\
x_6 & = & & x_5 & -4x_2 & -2x_3 & +8x_4 \\
x_7 & = & 1 & +2x_5 & -11x_2 & -5x_3 & +18x_4 \\
z & = & & -20x_5 & +53x_2 & +41x_3 & -204x_4
\end{array}
$$

x_2 enters and x_6 leaves

dictionary 3

$$
\begin{array}{rcrrrrr}
x_1 & = & & +.75x_5 & -2.75x_6 & -.5x_3 & +4x_4 \\
x_2 & = & & .25x_5 & -.25x_6 & -.5x_3 & +2x_4 \\
x_7 & = & 1 & -.75x_5 & -13.25x_6 & +.5x_3 & -4x_4 \\
z & = & & -6.75x_5 & -13.25x_6 & +14.5x_3 & -98x_4
\end{array}
$$

LP Cycling Example

dictionary 3

$$\begin{array}{rcrrrrr} x_1 & = & & +.75x_5 & -2.75x_6 & -.5x_3 & +4x_4 \\ x_2 & = & & .25x_5 & -.25x_6 & -.5x_3 & +2x_4 \\ x_7 & = & 1 & -.75x_5 & -13.25x_6 & +.5x_3 & -4x_4 \\ z & = & & -6.75x_5 & -13.25x_6 & +14.5x_3 & -98x_4 \end{array}$$

LP Cycling Example

dictionary 3

$$\begin{array}{rcrrrrr} x_1 & = & & +.75x_5 & -2.75x_6 & -.5x_3 & +4x_4 \\ x_2 & = & & .25x_5 & -.25x_6 & -.5x_3 & +2x_4 \\ x_7 & = & 1 & -.75x_5 & -13.25x_6 & +.5x_3 & -4x_4 \\ z & = & & -6.75x_5 & -13.25x_6 & +14.5x_3 & -98x_4 \end{array}$$

x_3 enters

LP Cycling Example

dictionary 3

$$\begin{array}{rcrrrrr} x_1 & = & & +.75x_5 & -2.75x_6 & -.5x_3 & +4x_4 \\ x_2 & = & & .25x_5 & -.25x_6 & -.5x_3 & +2x_4 \\ x_7 & = & 1 & -.75x_5 & -13.25x_6 & +.5x_3 & -4x_4 \\ z & = & & -6.75x_5 & -13.25x_6 & +14.5x_3 & -98x_4 \end{array}$$

x_3 enters and x_1 leaves

LP Cycling Example

dictionary 3

$$\begin{array}{rclrrrr} x_1 & = & & +.75x_5 & -2.75x_6 & -.5x_3 & +4x_4 \\ x_2 & = & & .25x_5 & -.25x_6 & -.5x_3 & +2x_4 \\ x_7 & = & 1 & -.75x_5 & -13.25x_6 & +.5x_3 & -4x_4 \\ z & = & & -6.75x_5 & -13.25x_6 & +14.5x_3 & -98x_4 \end{array}$$

x_3 enters and x_1 leaves

dictionary 4

$$\begin{array}{rclrrrr} x_3 & = & & +.1.5x_5 & -5.5x_6 & -2x_1 & +8x_4 \\ x_2 & = & & -.5x_5 & +2.5x_6 & +x_1 & -2x_4 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 15x_5 & -93x_6 & -29x_1 & +18x_4 \end{array}$$

LP Cycling Example

dictionary 4

$$
\begin{array}{rclrrrr}
x_3 & = & & +.1.5x_5 & -5.5x_6 & -2x_1 & +8x_4 \\
x_2 & = & & -.5x_5 & +2.5x_6 & +x_1 & -2x_4 \\
x_7 & = & 1 & & & -x_1 & \\
z & = & & 15x_5 & -93x_6 & -29x_1 & +18x_4
\end{array}
$$

LP Cycling Example

dictionary 4

$$\begin{array}{rcrrrrr} x_3 & = & & +.1.5x_5 & -5.5x_6 & -2x_1 & +8x_4 \\ x_2 & = & & -.5x_5 & +2.5x_6 & +x_1 & -2x_4 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 15x_5 & -93x_6 & -29x_1 & +18x_4 \end{array}$$

x_4 enters

LP Cycling Example

dictionary 4

$$\begin{array}{rcrrrrr} x_3 & = & & +.1.5x_5 & -5.5x_6 & -2x_1 & +8x_4 \\ x_2 & = & & -.5x_5 & +2.5x_6 & +x_1 & -2x_4 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 15x_5 & -93x_6 & -29x_1 & +18x_4 \end{array}$$

x_4 enters and x_2 leaves

LP Cycling Example

dictionary 4

$$\begin{array}{rclrrrr} x_3 & = & & +.1.5x_5 & -5.5x_6 & -2x_1 & +8x_4 \\ x_2 & = & & -.5x_5 & +2.5x_6 & +x_1 & -2x_4 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 15x_5 & -93x_6 & -29x_1 & +18x_4 \end{array}$$

x_4 enters and x_2 leaves

dictionary 5

$$\begin{array}{rclrrrr} x_3 & = & & -.5x_5 & +4.5x_6 & +2x_1 & -4x_2 \\ x_4 & = & & -.25x_5 & +1.25x_6 & +.5x_1 & -.5x_2 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 10.5x_5 & -70.5x_6 & -20x_1 & -9x_2 \end{array}$$

LP Cycling Example

dictionary 5

$$
\begin{array}{rclrrrr}
x_3 & = & & -.5x_5 & +4.5x_6 & +2x_1 & -4x_2 \\
x_4 & = & & -.25x_5 & +1.25x_6 & +.5x_1 & -.5x_2 \\
x_7 & = & 1 & & & -x_1 & \\
z & = & & 10.5x_5 & -70.5x_6 & -20x_1 & -9x_2
\end{array}
$$

LP Cycling Example

dictionary 5

$$\begin{array}{rcrrrrr} x_3 & = & & -.5x_5 & +4.5x_6 & +2x_1 & -4x_2 \\ x_4 & = & & -.25x_5 & +1.25x_6 & +.5x_1 & -.5x_2 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 10.5x_5 & -70.5x_6 & -20x_1 & -9x_2 \end{array}$$

x_5 enters

LP Cycling Example

dictionary 5

$$\begin{array}{rclrrrr} x_3 & = & & -.5x_5 & +4.5x_6 & +2x_1 & -4x_2 \\ x_4 & = & & -.25x_5 & +1.25x_6 & +.5x_1 & -.5x_2 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 10.5x_5 & -70.5x_6 & -20x_1 & -9x_2 \end{array}$$

x_5 enters and x_3 leaves

LP Cycling Example

dictionary 5

$$\begin{aligned}
x_3 &= & -.5x_5 & +4.5x_6 & +2x_1 & -4x_2 \\
x_4 &= & -.25x_5 & +1.25x_6 & +.5x_1 & -.5x_2 \\
x_7 &= 1 & & & -x_1 & \\
z &= & 10.5x_5 & -70.5x_6 & -20x_1 & -9x_2
\end{aligned}$$

x_5 enters and x_3 leaves

dictionary 6

$$\begin{aligned}
x_5 &= & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\
x_4 &= & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\
x_7 &= 1 & & & -x_1 & \\
z &= & -21x_3 & +24x_6 & +22x_1 & -93x_2
\end{aligned}$$

LP Cycling Example

dictionary 6

$$
\begin{array}{rclrrrr}
x_5 & = & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\
x_4 & = & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\
x_7 & = & 1 & & & -x_1 & \\
z & = & & -21x_3 & +24x_6 & +22x_1 & -93x_2
\end{array}
$$

LP Cycling Example

dictionary 6

$$\begin{array}{rcrrrrr} x_5 & = & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\ x_4 & = & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & -21x_3 & +24x_6 & +22x_1 & -93x_2 \end{array}$$

x_6 enters

LP Cycling Example

dictionary 6

$$\begin{array}{rcrrrrr} x_5 & = & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\ x_4 & = & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & -21x_3 & +24x_6 & +22x_1 & -93x_2 \end{array}$$

x_6 enters and x_4 leaves

LP Cycling Example

dictionary 6

$$\begin{array}{rclcccc} x_5 & = & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\ x_4 & = & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & -21x_3 & +24x_6 & +22x_1 & -93x_2 \end{array}$$

x_6 enters and x_4 leaves

dictionary 7

$$\begin{array}{rclcccc} x_5 & = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\ x_6 & = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\ x_7 & = & 1 & -x_1 & & & \\ z & = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 \end{array}$$

LP Cycling Example

Thus Dictionary 7 is Dictionary 1 and so we have returned to dictionary 1 (which is not surprising since we have returned to the same basis $\{x_5, x_6, x_7\}$). We call this cycling since we would repeat this over and over ad infinitum if we continue following Anstee's rule.

Bland's Rule chooses the entering variable by choosing the variable of smallest subscript among all variables with positive coefficient in the objective function. Bland's Rule avoids cycling.

In our case in Dictionary 6, we would choose a different entering variable

LP Cycling Example: Bland's Rule

dictionary 6

$$
\begin{array}{rcrrrrr}
x_5 & = & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\
x_4 & = & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\
x_7 & = & 1 & & & -x_1 & \\
z & = & & -21x_3 & +24x_6 & +22x_1 & -93x_2
\end{array}
$$

LP Cycling Example: Bland's Rule

dictionary 6

$$
\begin{array}{rcrrrrr}
x_5 & = & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\
x_4 & = & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\
x_7 & = & 1 & & & -x_1 & \\
z & = & & -21x_3 & +24x_6 & +22x_1 & -93x_2
\end{array}
$$

x_1 enters (by Bland's Rule)

LP Cycling Example: Bland's Rule

dictionary 6

$$\begin{array}{rcrrrrr}
x_5 & = & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\
x_4 & = & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\
x_7 & = & 1 & & & -x_1 & \\
z & = & & -21x_3 & +24x_6 & +22x_1 & -93x_2
\end{array}$$

x_1 enters (by Bland's Rule) and x_4 leaves

LP Cycling Example: Bland's Rule

dictionary 6

$$\begin{aligned}
x_5 &= & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\
x_4 &= & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\
x_7 &= & 1 & & & -x_1 & \\
z &= & & -21x_3 & +24x_6 & +22x_1 & -93x_2
\end{aligned}$$

x_1 enters (by Bland's Rule) and x_4 leaves

dictionary 8

$$\begin{aligned}
x_5 &= & & +2x_3 & +x_6 & -8x_4 & +4x_2 \\
x_1 &= & & +x_3 & -2x_6 & -2x_4 & +3x_2 \\
x_7 &= & 1 & -x_3 & +2x_6 & +2x_4 & -3x_2 \\
z &= & & x_3 & -20x_6 & -44x_4 & -27x_2
\end{aligned}$$

LP Cycling Example: Bland's Rule

dictionary 8

$$
\begin{array}{rcccccc}
x_5 & = & & +2x_3 & +x_6 & -8x_4 & +4x_2 \\
x_1 & = & & +x_3 & -2x_6 & -2x_4 & +3x_2 \\
x_7 & = & 1 & -x_3 & +2x_6 & +2x_4 & -3x_2 \\
z & = & & x_3 & -20x_6 & -44x_4 & -27x_2
\end{array}
$$

LP Cycling Example: Bland's Rule

dictionary 8

$$\begin{array}{rcrrrrr} x_5 & = & & +2x_3 & +x_6 & -8x_4 & +4x_2 \\ x_1 & = & & +x_3 & -2x_6 & -2x_4 & +3x_2 \\ x_7 & = & 1 & -x_3 & +2x_6 & +2x_4 & -3x_2 \\ z & = & & x_3 & -20x_6 & -44x_4 & -27x_2 \end{array}$$

x_3 enters

LP Cycling Example: Bland's Rule

dictionary 8

$$
\begin{array}{rclrrrr}
x_5 & = & & +2x_3 & +x_6 & -8x_4 & +4x_2 \\
x_1 & = & & +x_3 & -2x_6 & -2x_4 & +3x_2 \\
x_7 & = & 1 & -x_3 & +2x_6 & +2x_4 & -3x_2 \\
z & = & & x_3 & -20x_6 & -44x_4 & -27x_2
\end{array}
$$

x_3 enters and x_7 leaves

LP Cycling Example: Bland's Rule

dictionary 8

$$\begin{array}{rcrrrrr} x_5 & = & & +2x_3 & +x_6 & -8x_4 & +4x_2 \\ x_1 & = & & +x_3 & -2x_6 & -2x_4 & +3x_2 \\ x_7 & = & 1 & -x_3 & +2x_6 & +2x_4 & -3x_2 \\ z & = & & x_3 & -20x_6 & -44x_4 & -27x_2 \end{array}$$

x_3 enters and x_7 leaves

dictionary 9

$$\begin{array}{rcrrrrr} x_5 & = & 2 & -2x_7 & +5x_6 & -4x_4 & -2x_2 \\ x_1 & = & 1 & -x_7 & & & \\ x_3 & = & 1 & -x_7 & +2x_6 & +2x_4 & -3x_2 \\ z & = & 1 & -x_7 & -18x_6 & -42x_4 & -30x_2 \end{array}$$

Optimal solution $(1, 0, 1, 0, 2, 0, 0)$ with $z = 1$.