Unbounded LP Example

Unbounded LP Example

$$
\begin{array}{lrrrrl}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -x_1 & +x_2 & -x_3 \\
x_5 & = & 3 & +2x_1 & -x_2 & \\
x_6 & = & 5 & & -x_2 & +2x_3 \\
z & = & & & 2x_2 & +x_3
\end{array}
$$

Unbounded LP Example

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\begin{array}{lrrrrl}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
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\end{array}
\qquad x_1, x_2, x_3 \geq 0
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\begin{array}{rcrrrr}
x_4 & = & 5 & -x_1 & +x_2 & -x_3 \\
x_5 & = & 3 & +2x_1 & -x_2 & \\
x_6 & = & 5 & & -x_2 & +2x_3 \\
z & = & & & 2x_2 & +x_3
\end{array}
$$

x_2 enters

Unbounded LP Example

$$
\begin{array}{lrrrrl}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -x_1 & +x_2 & -x_3 \\
x_5 & = & 3 & +2x_1 & -x_2 & \\
x_6 & = & 5 & & -x_2 & +2x_3 \\
z & = & & & 2x_2 & +x_3
\end{array}
$$

x_2 enters and x_5 leaves

Unbounded LP Example

$$
\begin{array}{lrrrrl}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

$$
\begin{array}{rcrrrr}
x_4 & = & 5 & -x_1 & +x_2 & -x_3 \\
x_5 & = & 3 & +2x_1 & -x_2 & \\
x_6 & = & 5 & & -x_2 & +2x_3 \\
z & = & & & 2x_2 & +x_3
\end{array}
$$

x_2 enters and x_5 leaves

$$
\begin{array}{rcrrrr}
x_4 & = & 8 & +x_1 & -x_5 & -x_3 \\
x_2 & = & 3 & +2x_1 & -x_5 & \\
x_6 & = & 2 & -2x_1 & +x_5 & +2x_3 \\
z & = & 6 & +4x_1 & -2x_5 & +x_3
\end{array}
$$

Unbounded LP Example

$$
\begin{array}{lrrrrl}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5 \\
x_4 & = & 8 & +x_1 & -x_5 & -x_3 \\
x_2 & = & 3 & +2x_1 & -x_5 & \\
x_6 & = & 2 & -2x_1 & +x_5 & +2x_3 \\
z & = & 6 & +4x_1 & -2x_5 & +x_3
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

Unbounded LP Example

$$
\begin{array}{lrrrrl}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

$$
\begin{array}{rcrrrr}
x_4 & = & 8 & +x_1 & -x_5 & -x_3 \\
x_2 & = & 3 & +2x_1 & -x_5 & \\
x_6 & = & 2 & -2x_1 & +x_5 & +2x_3 \\
z & = & 6 & +4x_1 & -2x_5 & +x_3
\end{array}
$$

x_1 enters

Unbounded LP Example

$$
\begin{array}{lrrrrr}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

$$
\begin{array}{rcrrrr}
x_4 & = & 8 & +x_1 & -x_5 & -x_3 \\
x_2 & = & 3 & +2x_1 & -x_5 & \\
x_6 & = & 2 & -2x_1 & +x_5 & +2x_3 \\
z & = & 6 & +4x_1 & -2x_5 & +x_3
\end{array}
$$

x_1 enters and x_6 leaves

Unbounded LP Example

$$
\begin{array}{lrrrrr}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

$$
\begin{array}{rcrrrr}
x_4 & = & 8 & +x_1 & -x_5 & -x_3 \\
x_2 & = & 3 & +2x_1 & -x_5 & \\
x_6 & = & 2 & -2x_1 & +x_5 & +2x_3 \\
z & = & 6 & +4x_1 & -2x_5 & +x_3
\end{array}
$$

x_1 enters and x_6 leaves

$$
\begin{array}{rcrrrr}
x_4 & = & 9 & -\frac{1}{2}x_6 & -\frac{1}{2}x_5 & \\
x_2 & = & 5 & -x_6 & & +2x_3 \\
x_1 & = & 1 & -\frac{1}{2}x_6 & +\frac{1}{2}x_5 & +x_3 \\
z & = & 10 & -2x_6 & & +5x_3
\end{array}
$$

Unbounded LP Example

$$
\begin{array}{lrrrrrr}
\max & & 2x_2 & +x_3 & & & \\
 & x_1 & -x_2 & & \leq & 5 & \\
 & -2x_1 & +x_2 & & \leq & 3 & \\
 & & x_2 & -2x_3 & \leq & 5 & \\
x_4 & = & 9 & -\frac{1}{2}x_6 & -\frac{1}{2}x_5 & & \\
x_2 & = & 5 & -x_6 & & +2x_3 & \\
x_1 & = & 1 & -\frac{1}{2}x_6 & +\frac{1}{2}x_5 & +x_3 & \\
z & = & 10 & -2x_6 & & +5x_3 &
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

Unbounded LP Example

$$
\begin{array}{lrrrrrr}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5 \\
x_4 & = & 9 & -\frac{1}{2}x_6 & -\frac{1}{2}x_5 & \\
x_2 & = & 5 & -x_6 & & +2x_3 \\
x_1 & = & 1 & -\frac{1}{2}x_6 & +\frac{1}{2}x_5 & +x_3 \\
z & = & 10 & -2x_6 & & +5x_3
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

x_3 enters

Unbounded LP Example

$$
\begin{array}{lrrrrl}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

$$
\begin{array}{lcrrrr}
x_4 & = & 9 & -\frac{1}{2}x_6 & -\frac{1}{2}x_5 & \\
x_2 & = & 5 & -x_6 & & +2x_3 \\
x_1 & = & 1 & -\frac{1}{2}x_6 & +\frac{1}{2}x_5 & +x_3 \\
z & = & 10 & -2x_6 & & +5x_3
\end{array}
$$

x_3 enters and no leaving variable (no restriction on increase to x_3)

Unbounded LP Example

$$
\begin{array}{lrrrrrl}
\max & & 2x_2 & +x_3 & & & \\
 & x_1 & -x_2 & & \leq & 5 & \\
 & -2x_1 & +x_2 & & \leq & 3 & x_1, x_2, x_3 \geq 0 \\
 & & x_2 & -2x_3 & \leq & 5 & \\
x_4 & = & 9 & -\frac{1}{2}x_6 & -\frac{1}{2}x_5 & & \\
x_2 & = & 5 & -x_6 & & +2x_3 & \\
x_1 & = & 1 & -\frac{1}{2}x_6 & +\frac{1}{2}x_5 & +x_3 & \\
z & = & 10 & -2x_6 & & +5x_3 &
\end{array}
$$

x_3 enters and no leaving variable (no restriction on increase to x_3)

Parametric solution showing that LP is unbounded:

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$$
\begin{array}{lrrrrl}
\max & & 2x_2 & +x_3 & & \\
 & x_1 & -x_2 & & \leq & 5 \\
 & -2x_1 & +x_2 & & \leq & 3 \\
 & & x_2 & -2x_3 & \leq & 5
\end{array}
\qquad x_1, x_2, x_3 \geq 0
$$

$$
\begin{array}{rcrrrr}
x_4 & = & 9 & -\frac{1}{2}x_6 & -\frac{1}{2}x_5 & \\
x_2 & = & 5 & -x_6 & & +2x_3 \\
x_1 & = & 1 & -\frac{1}{2}x_6 & +\frac{1}{2}x_5 & +x_3 \\
z & = & 10 & -2x_6 & & +5x_3
\end{array}
$$

x_3 enters and no leaving variable (no restriction on increase to x_3)

Parametric solution showing that LP is unbounded:

$$
\begin{array}{rcrr}
x_1 & = & 1 & +t \\
x_2 & = & 5 & +2t \\
x_3 & = & & t \\
x_4 & = & 9 &
\end{array}
$$

with $x_5 = x_6 = 0$ and $z = 10 + 5t$ for $t \geq 0$.