Illumination with Orthogonal Floodlights* (Extended Abstract)

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Abstract. We provide the first tight bound for covering a polygon with n vertices and h holes with vertex guards. In particular, we provide tight bounds for the number of floodlights, placed at vertices or on the boundary, sufficient to illuminate the interior or the exterior of an orthogonal polygon with holes. Our results lead directly to simple linear, and thus optimal, algorithms for computing a covering of an orthogonal polygon.

1 Introduction

The question of guarding a polygonal art gallery has raised many problems ranging from polygon decomposition and problem complexity to combinatorial structure of visibility graphs [1, 11]. Moreover, the study of visibility in this type of geometric setting has not only been naturally motivated by many applications, but it has also been fundamental in developing many theoretical and practical results [13]. Despite the many variants of the problem, little regard has been placed to the assumption that guards can cover a complete 2π range of orientations around them. Rawlins [12] studied visibility along finitely oriented staircases and provided corresponding Art Gallery Theorems. However, only recently the question of studying visibility covers with *floodlights* (that is, static guards with restricted angle of vision) has been raised. The problem of covering a line with floodlights has been labeled the *stage illumination problem* [3, 5] while the problem of covering the plane has also been discussed [3, 14].

Observe that in O'Rourke's book [11] the first sentence of Chapter 5 says "One of the major open problems in the field of art gallery theorems is to establish a theorem for polygons with holes". In this paper we extend the tight bound of $\lfloor 3(n-1)/8 \rfloor$ [6] for the number of orthogonal floodlights sufficient to cover an orthogonal polygon with n vertices to the case of polygons with holes. In particular, we show that $\lfloor (3n + 4(h-1))/8 \rfloor$ orthogonal floodlights placed

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at vertices are always sufficient and sometimes necessary to cover an orthogonal polygon with n vertices and h holes. Moreover, we show that the above bound is reduced to $\lfloor (n+2h)/4 \rfloor$ if the orthogonal floodlights may be placed in the boundary of the polygon. Hoffman [8] proved the first tight bound previous to these results; namely, $\lfloor n/4 \rfloor$ guards are sometimes necessary and always sufficient to cover an orthogonal polygon independent of the number of holes. However, Hoffmann's results require that the guards be placed on points other than the vertices or the boundary. Hoffman also claims an $O(n^{1.5} \log^2 \log \log n)$ guard placement algorithm. Two groups independently [2] proved that $\lfloor (n+h)/3 \rfloor$ is the corresponding tight bound for general polygons; but again, guards must be allowed in the interior of the polygon. Other efforts to provide tight bounds for general polygons with holes and orthogonal polygons with holes have been successful for only special cases [7, 13]. Thus, our results provide the first theorem with a tight bound for vertex guarding polygons with holes, for all values of n and h. Moreover, our proofs lead to linear algorithms that are simple and practical since they avoid complex polygon decomposition (we do not need to triangulate or to find a quadrilateralization of the polygon). We observe that, for the case $h = 0$, the original art gallery theorem requires long proofs [10], however, our proofs are simple.

Moreover, from the point of view of total aperture used, the original art gallery theorems may require an aperture of $3\pi/2$ for each of the $\lfloor n/4 \rfloor$ floodlights resulting in a total aperture of $3n\pi/8$ (in fact, Fig. 2.12 of O'Rourke's book [11] illustrates a possible worst case where all floodlights that the algorithm finds must be of aperture $3\pi/2$). This can be regarded as unnecessarily inefficient, since for example, for polygons without holes our results demonstrate that only $\lfloor (3n-4)/8 \rfloor$ floodlights, each of aperture $\pi/2$, are needed even if we are required to place them at vertices. The total aperture obtained with our algorithms is half the naive use of the original art gallery theorem and corresponding algorithms.

In Section 2 we demonstrate that an aperture of $\pi/2$ is necessary for vertex floodlights to illuminate an orthogonal polygon. We also show that $\lfloor (3n + 4(h-1))/8 \rfloor$ orthogonal floodlights are sometimes necessary. Section 3 proves that $\lfloor (3n + 4(h-1))/8 \rfloor$ orthogonal floodlights placed at vertices are always sufficient and describes the linear algorithm to find the covering. Section 4 discusses the case where the floodlights are placed on the boundary. We show that $\lfloor (n+2h)/4 \rfloor$ floodlights are always sufficient and sometimes necessary. Section 5 provides an Art Gallery Theorem and a corresponding linear algorithm for illuminating the exterior of a simply-connected orthogonal polygon with holes; namely that $(n+4)/2$ orthogonal floodlights are always sufficient and sometimes necessary. Section 6 provides some final remarks.

2 Necessity Bounds

Consider a polygonal art gallery given by an orthogonal polygon P in the plane with n vertices and h holes. There are n_0 vertices on the outer boundary and n_i vertices in the i-th hole, with $\sum_{i=0}^{h} n_i = n$. An α-floodlight is a lamp that shines light in a wedge of aperture α. We are interested in determining a set of α-floodlights that illuminate (cover) the interior or the exterior of P. If we require that the floodlights be placed at vertices, we will call them vertex floodlights, or v-floodlights for short. Note that no more than one floodlight is allowed at a point in P, otherwise, we have one floodlight of larger aperture. The following result demonstrates that an aperture of $\pi/2$ is the smallest aperture necessary for orthogonal polygons.

Theorem 1. *For all $\epsilon > 0$, there is an orthogonal polygon that can not be illuminated with vertex floodlights in each vertex with aperture $\frac{\pi}{2} - \epsilon$.*

Proof. Let λ be a line segment with endpoints a and b and middle point in the origin O. Let the slope of λ be $\delta < \epsilon$ and $\theta = \frac{\pi}{2} - \delta$. Let λ' be a segment orthogonal to λ with the same length as λ and also with middle point in the origin, but with endpoints c and d. We construct an orthogonal polygon with 12 vertices in the shape of an helix and whose reflex vertices are precisely a, b, c and d; refer to Fig. 1. If the prongs of P are large enough, the line of slope

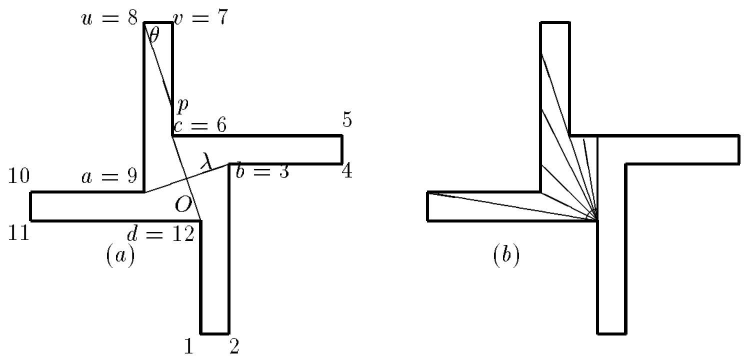

Fig. 1. This orthogonal helix requires four orthogonal v-floodlights.

$\theta + \pi/2$ through u intersects the side cv in a point p far enough from c; see Fig. 1 again. The reader can now verify that P can not be illuminated with floodlights of aperture $\frac{\pi}{2} - \epsilon$ in each vertex. □

In what follows we concentrate on floodlights with aperture $\pi/2$; we will call them orthogonal floodlights. The helix of Fig. 1 will provide the building block to prove a bound of the number of orthogonal floodlights necessary to illuminate an orthogonal polygon with n vertices and h holes. First, we will prove the bound for the case $h = 0$, that is polygons without holes, and later the general case.

Consider again the orthogonal polygon in Fig. 1, where the prongs are long enough that a floodlight as shown in part (b) can illuminate at most one prong.

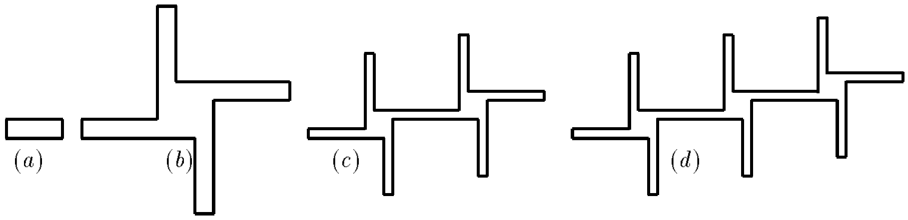

Fig. 2. Orthogonal polygons that require one $\pi/2$-floodlight for each prong.

Clearly, at least 4 orthogonal floodlights are required to cover this polygon. Now, consider the progression illustrated by Fig. 2. At each new stage, we merge a copy of the polygon in Fig. 1 by its left prong. Vertices 10 and 11 are identified with the two right-most vertices in the previous figure. It is not hard to see that at each stage, three more orthogonal v-floodlights are needed but only eight vertices are added. It now follows that $\lfloor(3n-4)/8\rfloor$ floodlights are necessary.

We now provide polygons that demonstrate the necessity of $\lfloor\frac{3n+4(h-1)}{8}\rfloor$ orthogonal floodlights. Consider the polygon P_{32} of Fig. 3 (a). It has 32 vertices and one hole. This polygon is constructed with four copies of the helix of Fig. 1 joined to form a polygon with one hole. Moreover, P_{32} requires 12 orthogonal floodlights, one for each of the eight prongs and one for each of the four alleys. This polygon provides the basic building block. For larger values of h, we join the the right-most edge of one copy of P_{32} with the left-most edge of another. For example, Fig. 3 (b) is a polygon with 60 vertices and two holes that requires 23 orthogonal floodlights. The process can be repeated to generate polygons with $32+28(h-1)$ vertices and h holes that need $12+11(h-1)$ orthogonal floodlights. Other values of n can be obtained eliminating the extra prongs.

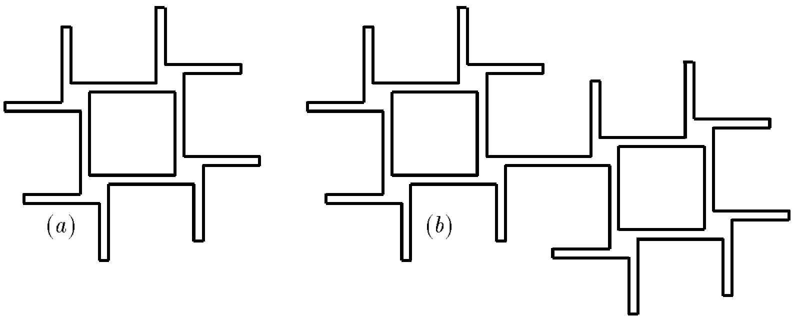

Fig. 3. Polygons that require one $\pi/2$-floodlight for each prong or alley.

3 Illuminating Vertex Floodlights

We now prove that $\lfloor(3n+4(h-1))/8\rfloor$ orthogonal v-floodlights are always sufficient to illuminate an orthogonal art gallery with n vertices and h holes. We use the following notation introduced by Rawlins [12]. Given an orthogonal polygon P, an edge e of P is said to be a *North edge* (N-edge for short), if the interior of the polygon is immediately below e. East, West and South edges are defined analogously. A vertex is said to be a *North-East vertex* (NE-vertex for short), if the polygonal edges that intersect at the vertex are an N-edge and an E-edge. NW-vertices, SE-vertices and SW-vertices are defined similarly. Since a vertex may also be convex or reflex, there are eight possible types of vertices in an orthogonal polygon. We define the following floodlight placement rule.

Definition 2. *North-East rule* (NE-rule): For each North edge e of the polygon, place a floodlight aligned with e at the East vertex of e. For each East edge e of the polygon, place a floodlight aligned with e at the North vertex of e. For a diagram of the NE-rule see Fig. 4 (a).

Lemma 3. *Let P be an orthogonal polygon with holes. The NE-rule produces an assignment of floodlights that illuminates the interior of P.*

Proof. Let p be a point in the interior of P. Let x be the first point in the border of P visible by a horizontal ray from p to the East. Clearly, x is in an E-edge e and p is visible from x. Consider a point x' in e just above x and consider the rectangle R with extreme points at x' and p; see Fig. 4 (b). Clearly, if x' is close enough to x, the rectangle R is contained in P. Consider moving x' North until it cannot be moved further without R leaving P. This happens because

- x' has reached the North vertex of e, in which case, p is illuminated by a floodlight at this point; see Fig. 4 (c), or
- the upper side of the rectangle R has coincided with a North edge, in which case, p is illuminated by a floodlight at the East point of this North edge; see Fig. 4 (d).

In both cases, p is illuminated and the proof is complete. □

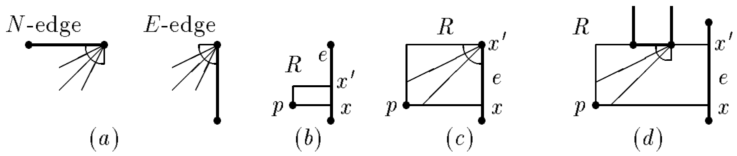

Fig. 4. Diagram illustrating the placement of floodlights by the NE-rule

Similarly, we can define a NW-rule, a SE-rule and a SW-rule, each illuminating the polygon. We are now ready to prove sufficiency.

Lemma 4. *Let P be an orthogonal polygon with n vertices and h holes. A total of $\lfloor (3n+4(h-1))/8 \rfloor$ orthogonal floodlights are sufficient to illuminate P.*

Proof. Illuminate the polygon P by each of the four rules proposed above. Let $\|X\|$ denote the number of floodlights used by the X rule. Note that each edge of the polygon receives at most two floodlights (for example, a N-edge receives a floodlight at its E-vertex in the NE-rule and at its W-vertex in the NW-rule) and the sets of floodlights of any pair of rules is disjoint. Moreover, in the NE-rule, a NE-convex vertex receives only one floodlight. Thus, the number $\|NE\|$ of floodlights used by the NE-rule is given by

$$\|NE\| = \|SE\|_r + \|NW\|_r + \|NE\|_c,$$

where $\|SE\|_r$ is the number of SE-reflex vertices, $\|NW\|_r$ is the number of NW-reflex vertices, and $\|NE\|_c$ is the number of NE-convex vertices. Thus, the total number of orthogonal floodlights used by the four rules is given by $\|NE\| + \|NW\| + \|SE\| + \|SW\| = 2r + c$, where r is the number of reflex vertices in the polygon P and c is the number of convex vertices. Since for an orthogonal polygon with no holes $c = (n+4)/2$ [11] and $r = (n-4)/2$ [11], and also, for a polygon with holes, the convex vertices on a hole are reflex vertices for the hole, while the reflex vertices on a hole are convex vertices for the hole, we have that the covering rule that uses the minimum number of floodlights uses

$$\left\lfloor \frac{2r+c}{4} \right\rfloor = \left\lfloor \frac{2(r_0 + c_1 + \ldots + c_h) + c_0 + r_1 + \ldots + r_h}{4} \right\rfloor$$

floodlights, where $c_i + r_i = n_i$ and c_i is the number of convex vertices in the i-th hole. Since $n = \sum_{n=0}^{h} n_i$, we have that $\lfloor (2r+c)/4 \rfloor$ is

$$\left\lfloor \frac{2\left[\frac{n_0-4}{2} + \sum_{i=1}^{h} \frac{n_i+4}{2}\right] + \frac{n_0+4}{2} + \sum_{i=1}^{h} \frac{n_i-4}{2}}{4} \right\rfloor = \left\lfloor \frac{3n+4(h-1)}{8} \right\rfloor.$$

This completes the proof. □

We have proved the following result.

Theorem 5. *If P is an orthogonal polygon with n vertices and h holes, then $\lfloor (3n+4(h-1))/8 \rfloor$ orthogonal floodlights are always sufficient and sometimes necessary to illuminate P.*

We claim that this result is significant despite the fact that it may suggest that more guards are required than in the original art gallery theorem. We support this claim with three observations:

1.- Using Theorem 5, the total aperture of $3\pi n/8$ proposed by the original version of the orthogonal art gallery theorem, for polygons without holes, has been reduced by half. Moreover, for polygons with holes, the total aperture using floodlights is always less than the $\pi n/2$ proposed by Hoffmann's result.

2.- The placing rules lead directly to a linear algorithm that is much simpler than the algorithms for guards that require trapezoidization, quadrilateralization or decomposition into L-shaped pieces [11]. The algorithm consists of a traversal of the boundary of the polygon that counts the types of vertices. It computes the number of vertex floodlights required by each of the four rules, determines which one uses the minimum and assigns floodlights in the corresponding vertices with a second traversal of the boundary.
3.- The algorithm may place many fewer floodlights that $\lfloor(3n + 4(h-1))/8\rfloor$; for example, in a staircase polygon, only one floodlight is used.

4 Illuminating with Floodlights on the Boundary

An orthogonal art gallery P with no holes and r reflex vertices can be partitioned into $\lfloor r/2\rfloor + 1$ L-shaped pieces [11], and since each L-shaped piece can be illuminated with one floodlight, we have that $\lfloor n/4\rfloor$ orthogonal floodlights are sometimes necessary and always sufficient to illuminate an orthogonal polygon. This seems to contradict our previous results; however, the missing detail is that using O'Rourke's algorithm to partition P into L-shaped pieces, some of the floodlights will be placed in the interior of the polygon. This seems rather unsatisfactory.

In this section, we first show that we can illuminate a polygon P with no holes with $\lfloor n/4\rfloor$ orthogonal floodlights placed at points in the boundary of P. We prove this by showing that $\lfloor r/2\rfloor + 1$ orthogonal floodlights at points in the boundary are always sufficient to illuminate P, where r is the number of reflex vertices in P. Then, we demonstrate that any orthogonal polygon with n vertices and h holes can be illuminated with $\lfloor(n + 2h)/4\rfloor$ floodlights in its boundary.

For a polygon with no holes, necessity of $\lfloor n/4\rfloor$ floodlights is established by the well-known "comb" example [11, Figure 2.18]. Sufficiency follows an inductive argument similar to O'Rourke's proof of the orthogonal art gallery theorem [11, Sections 2.5 and 2.6].

A *horizontal cut* of an orthogonal polygon P is an extension of the horizontal edge incident to a reflex vertex through the interior of the polygon. A cut *resolves* a reflex vertex in the sense that the vertex is no longer reflex in either of the two pieces of the partition determined by the cut. Clearly, a cut does not introduce any reflex vertices. A horizontal cut is an *odd-cut* (also and H-odd-cut) if one of the halves contains an odd number of reflex vertices.

Lemma 6. *Let P be an orthogonal polygon and partition P into $P_1, P_2, \ldots, P_t$ by drawing all H-odd-cuts and all H-cuts that are visibility rays of two reflex vertices. Then, each P_i is in general horizontal position and can be covered with $\lfloor r_i/2\rfloor + 1$ floodlights in the boundary of P, where r_i is the number of reflex vertices in P_i.*

In this extended abstract we omit the proof. Since the H-graph can be constructed in linear time [11] and trapezoidization can be achieved in linear time [4], it is not hard to see that the above argument results in a linear algorithm. Thus, we have obtained the following result.

Theorem 7. *Let P be an orthogonal polygon with n vertices, then $\lfloor n/4 \rfloor$ orthogonal floodlights placed in the boundary of P are always sufficient and sometimes necessary to illuminate the interior of P. Moreover, such set of floodlights can be found in $O(n)$ time.*

We are now ready to discuss the case of orthogonal polygons with holes.

Theorem 8. *An orthogonal polygon with n vertices and h holes can be illuminated with at most $\lfloor \frac{n+2h}{4} \rfloor$ floodlights in its boundary. Moreover, this bound is tight, since there are orthogonal polygons with n vertices and h holes that require $\lfloor \frac{n+2h}{4} \rfloor$ floodlights.*

Proof. To prove sufficiency, let P be a polygon with n vertices and h holes. Resolve all holes of P and construct a polygon P' as follows. Let v_0 be a reflex vertex that belongs to a hole P_0. Cut P along a horizontal segment λ in the interior of P that joins v_0 with the boundary of P. Include λ as two segments of the boundary. By Theorem 7, the polygon P' can be illuminated with $\lfloor (n + 2h)/4 \rfloor$ floodlights in its boundary. However, the cuts to construct P' from P were always horizontal, and the $\lfloor (n+2h)/4 \rfloor$ floodlights provided by Theorem 7 are placed on vertical edges of P'. Thus, these floodlights are on vertical edges of P. Moreover, it is not hard to see that the floodlights illuminate P. To prove necessity, we consider the generic polygon of Fig. 5. For each $m > 0$, this polygon

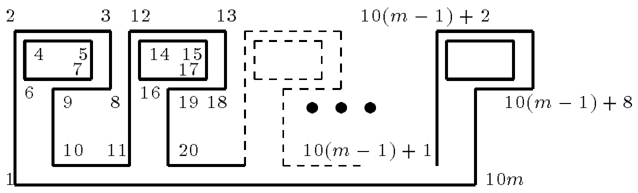

Fig. 5. A polygon with m holes and $10m$ vertices that requires $3m$ $\pi/2$-floodlights.

can be configured to have m holes and $10m$ vertices and requires $3m$ orthogonal floodlights. □

5 Illuminating the Exterior

Two other variants of the illumination problem are "The Fortress Problem" and the "Prison Yard Problem". The first asks for the number of floodlights to illuminate the exterior of the polygon, while the second asks for the number needed to see both the interior and the exterior.

In this section we demonstrate that $(n+4)/2$ orthogonal floodlights are sometimes necessary and always sufficient to illuminate the exterior of an orthogonal polygon P with n vertices and even with h holes.

We prove sufficiency first. Let P be an orthogonal polygon with n vertices and h holes. Again, let n_0 be the number of vertices in the outer boundary and n_i the number of vertices in the i-th hole. Illuminating the exterior of P requires us to illuminate the interior of the holes. Since P is simply-connected, the holes do not have holes of their own, thus, by the results of earlier sections, the i-th hole can be illuminated with fewer than $n_i/2$ orthogonal floodlights (recall that n_i must be even for $i = 0, \ldots, h$).

If we prove that the exterior of the polygon P' defined by the n_0 vertices of the outer boundary of P can be illuminated with $n_0/2 + 2$ floodlights, then we would have shown that $2 + \sum_{i=0}^{h} n_i/2 = 2 + n/2$ floodlights are sufficient.

Recall that a set S in the plane is orthogonally convex if any horizontal or vertical line segment intersects S in a connected set. The orthogonal convex hull $C(S)$ of a set S is the smallest orthogonally convex set containing S. Consider the orthogonal convex hull of P'. The boundary of $C(P')$ is composed of at most four staircases (and possibly fewer). Moreover, if this hull $C(P')$ has k vertices, its exterior can be illuminated with $k/2 + 2$ vertices by traveling around the boundary and placing one floodlight in every other vertex of each staircase and one on each of the North-most, South-most, East-most and West-most edges. Observe also that, for each new vertex introduced to the boundary of $C(P')$, a vertex is resolved from P' and no floodlights are assigned to a resolved vertex.

It only remains to illuminate the bays of P'; that is the regions exterior to P' but interior to the hull $C(P')$. We can not apply Theorem 5 directly and conclude that a bay with b vertices can be illuminated with $\lfloor (3b-4)/8 \rfloor$ orthogonal floodlights since a bay may have a vertex not originally in P'. However, estimating the size of the second smallest floodlight class in Theorem 5 suffices. Necessity is proved by an orthogonally convex polygon.

Theorem 9. *If P is an orthogonal polygon with n vertices and h holes, then $n/2+2$ orthogonal floodlights (vertex or at the boundary) are sometimes necessary and always sufficient to illuminate the exterior of P.*

Observe that the proof of this theorem leads again to a linear algorithm that does not require polygon partitioning and thus is simple and practical.

For the Prison Yard Problem, a rectangle illustrates that there is no solution for vertex floodlights unless we allow two floodlights to be placed at one vertex.

6 Concluding Remarks

We have established combinatorial tight bounds for illuminating the interior or exterior of orthogonal polygons with holes by orthogonal floodlights. Our proofs are simple and lead to linear algorithms on the number n of vertices of the polygon. The algorithms compute a covering achieving the bounds, and since any algorithm must inspect all points of the input, the computational complexity of our algorithms is optimal.

We have shown that $\pi/2$ is the smallest aperture necessary and in fact, it is not hard to see that for $\alpha \in [\pi/2, 3\pi/2)$ all our results hold for α-floodlights. It is trivial to note that for $\alpha \geq 3\pi/2$, α-floodlights are general vertex guards in orthogonal polygons. However, several open problems remain. What bounds can be found for other classes of polygons? Is computing the minimum set of covering α-floodlights an NP-hard or NP-complete problem? If the floodlights are each allowed to have a different aperture α_k, what can be said about the problem of finding a cover that optimizes the total angle power given by $\sum_{i=1}^{k} \alpha_k$?

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