CONTROL AND INVERSE PROBLEMS IN PARTIAL DIFFERENTIAL EQUATIONS

Huatulco, Oaxaca, Mexico
9 to 12 November, 2016

Within the context of developing scientific activities promoted by the Network of Mathematics and Development of CONACyT, this workshop aims to bring together applied mathematicians, to exchange ideas and promote research collaborations as well as to encourage student participation. The workshop will consist of four days of talks and three courses organised around different research topics on control theory and its applications as well as inverse problems and its impact in applied fields.

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Talks

1. Bayesian method for an inverse homogenized problem in low contrast conductivity.

Antonio Capella Kort. *IM, Universidad Nacional Autónoma de México*.

In this work we propose a Bayesian inversion approach for a problem in low contrast conductivity, combined in a homogenized conductivity problem. In this approach we combine ideas of the homogenization method in optimal design, as in the works of Allaire and Gutierrez and Gutierrez and Mura, with the Multi Chain Monte Carlo methods used in Bayesian approaches to inverse problems. The method as proposed can be also applied to problems where noise is added to some measurements.

2. Stable numerical solution of the Cauchy problem for the Laplace equation in a bounded annular region.

José Julio Conde Mones. *DM-DCBI, Universidad Autónoma Metropolitana - Iztapalapa*.
Co-authors: Lorenzo Héctor Juárez Valencia (DM-DCBI, UAM-Izt), José Jacobo Oliveros Oliveros (FCFM-BUAP) and María Monserrat Morín Castillo (FCE-BUAP).

This work presents a numerical study of Cauchy problem for the Laplace equation in a bounded annular region. To solve this ill-posed problem we follow a variational approach based on its reformulation as a boundary control problem, for which the cost function incorporates a penalized term with the input data. This functional is equivalent a the Tikhonov functional with parameter of regularization $\alpha = 1/k$, where k is the parameter of penalization. This functional is minimized by a conjugate gradient method in combination with a finite element discretization and where the regularization parameter is chosen using Tikhonov regularization method. Numerical solutions in simple and complex domains show that this methodology produces stable and accurate solutions.

3. On the Kalman rank condition for the controllability of a coupled system on n one dimensional wave equations.

Luz de Teresa. *IM, Universidad Nacional Autónoma de México*.
Co-authors: Sergei Avdonin.

In this conference we prove the exact controllability of a system of n one-dimensional wave equations when the control is exerted on a part of the boundary by means of one control. We give a Kalman condition (necessary and sufficient) and give a description, non optimal in general, of the attainable set.

4. Identification of piecewise constant sources in non-homogeneous media based on boundary measurements.

Andrés Fraguela Collar. *FCFM, Benemérita Universidad Autónoma de Puebla*.

In this study, we consider the inverse problem of determining a source term in an elliptic problem based on boundary measurements. The boundary measurements allow the unique determination of the harmonic component of the source, and thus a priori information is required for its complete identification. Using this a priori information, we determine the compactness of the class of sources and a uniqueness theorem for its identification from Cauchy data, thereby allowing us to propose a stable algorithm for finding "approximate" solutions of the inverse problem. The proposed procedure is demonstrated by individual cases, where we know the "geometry" of the inner region and the source takes a constant value in the inner region and another constant value on the rest of the domain.

References

[1] A. El Badia, T. Ha Duong. Some remarks on the problem of source identification from boundary measurements. *Inverse Prob.* 14 (1998) 883-891.

[2] Yu.M. Koptelov, E.V. Zakharov. Inverse problems in electroencephalography and their numerical solving, in: *A. Tikhonov (Ed.), Problems in Natural Sciences, VSP BV, Utrecht/TVP Sci. Publ., Moscow*, 1992, pp. 543-552. III-posed.

[3] A. Fraguela, M. Morín, J. Oliveros. Operational statement of the inverse problem of parameters localization of volumetric current in dipolo form (in Spanish). *Aportaciones Matemáticas, Serie Comunicaciones Sociedad Matemática Mexicana* 25 (1999) 41-55.

[4] A. Fraguela, J. Oliveros, M. Morín, L. Cervantes. Inverse electroencephalography for cortical sources. *Appl. Numer. Math.* 55 (2005) 191-203.

[5] A. Fraguela, M. Morín, J. Oliveros. Mathematical models in inverse electroencephalographic (in Spanish), in: *M.A. Jiménez Pozo, J. Bustamante González (Eds.), Tópicos de la Teoría de la Aproximación II, Textos Científicos. Benemérita Universidad Autónoma de Puebla*, 2007, pp. 73-95.

[6] A. Fraguela, M. Morín, J. Oliveros. Inverse electroencephalography for volumetric sources. *Math. Comput. Simul.* 78 (2008) 481-492.

[7] A. Fraguela, J. Oliveros , M. Morin, J. Conde. Identification of piecewise constant sources in non-homogeneous media based on boundary measurements. *Applied Mathematical Modelling* 39 (2015) 7697-7717.

5. Radial basis function method for solving convection diffusion constrained optimization problems.

Pedro González Casanova. *IM, Universidad Nacional Autónoma de México*.
Co-authors: Jorge Zavaleta Sánchez (FC, UNAM) and Jonnathan D. Rivera Ruiz (UNAM).

In this talk, and within the context of RBFs theory, we build and analyze different meshfree methods to solve convection diffusion problems for PDE-constrained optimization. Specifically, we introduce two new local RBFs methods, the first based on the combination of Local asymmetric technique with the Differential Quadrature method, LAM-DQ, and the second which uses two times the Local asymmetric method: LAM-LAM. Both techniques are used to solve several distributed control problems in order to compare their performance. We show that these local methods can attain errors which are comparable to the global collocation techniques, but using a computing time which is considerably faster. These local radial basis functions methods shows a possible way to solve massive problems of control optimization.

6. Solution of the inverse scattering problem from inhomogeneous media by affine invariant sampling.

Lili Guadarrama. *ConacytCIMAT Aguascalientes*.
Co-authors: M. Daza (CIMAT), M. A. Capistrán (CIMAT), J.A. Christen (CIMAT).

We propose in the Bayesian formulation of the inverse problem associated to recovering both the support and the refractive index of a convex obstacle given measurements of near-field scattered waves. Aiming at sampling efficiently from the arising posterior distribution using Markov Chain Monte Carlo, we construct a sampler, probability transition kernel, that is invariant under affine transformations of space.

To address the inverse scattering problem of time-harmonic acoustic waves by a penetrable nonhomogeneous medium and by a bounded penetrable obstacle of compact support at a fixed frequency, let consider as incident field a time-harmonic acoustic plane wave

$$u^i(\mathbf{x}, t) = e^{ik\mathbf{x}\cdot d}$$

where $k = w/c_0$ is the wave number, w frequency, c_0 the speed of sound and d the direction of propagation. We will consider the simplest scattering problem for the case of an nonhomogeneous medium, which consist to find u such that

$$
\begin{aligned}
\Delta u(\mathbf{x}) + k^2 b(\mathbf{x}) u(\mathbf{x}) &= 0 \ \ in \ \ \mathbb{R}^2 \\
u(\mathbf{x}) &= e^{ik\mathbf{x}\cdot d} + u^s(\mathbf{x}) \\
lim_{r\to\infty} r \left(\frac{\partial u^s(\mathbf{x})}{\partial r} - iku^s(\mathbf{x}) \right) &= 0
\end{aligned}
$$

where $r = |\mathbf{x}|$, $b(\mathbf{x}) : \mathbb{R}^2 \to \mathbb{C}$ is the refractive index, a piecewise constant function, $b(\mathbf{x}) = 1$ in the homogeneous host medium and we consider $b(\mathbf{x}) = constant = b \neq 1$ in $D \subset \mathbb{R}^2$, $u^s(\mathbf{x})$ is the scattered field. We are interested in recover ∂D and the refractive index $b(\mathbf{x})$ in D.

The formulation of the inverse problem in the Bayesian framework is

$$
\pi(\theta|u) = \frac{L(u^s|\theta)\pi(\theta)}{\pi(u^s)}. \tag{1}
$$

The posterior density $\pi(\theta|u^s)$ quantifies the uncertainty of the boundary and the refraction index $\theta = (\partial D, b)$ to be recovered from data u^s. The prior density $\pi(\theta)$ encodes the information we have of our unknown parameter using a probability distribution, and the likelihood $L(u|\theta)$ is an observational model which describes the relation between the observed and the unknown $\pi(u)$ is a normalizing constant. We use the point cloud affine invariant Markov Chain Monte Carlo Method sampling the posterior density [2].

References

[1] M.L. Daza, M.A. Capistrán, J.A. Christen, L. Guadarrama, *Solution of the inverse scattering problem from inhomogeneous media using affine invariant sampling.*, DOI: 10.1002/mma.3929

[2] A. Palafox, M.A. Capistrán, J.A. Christen, *Point cloud-based scatterer approximation and affine invariant sampling in the inverse scattering problem.*, DOI: 10.1002/mma.4056

7. Null controllability of a cascade system of Schrödinger equations.

Francisco Marcos López García. *IM-Cuernavaca, Universidad Nacional Autónoma de México*.

We present a control problem for a cascade system of two linear N-dimensional Schrödinger equations. We address the problem of null controllability by means of a control supported in a region not satisfying the classical geometrical control condition. This is a joint work with Alberto Mercado and Lucero de Teresa.

8. *Read/Write* operators for a Josephson Junction Array Memory: A controllability approach.

Jorge López López. *DCB, Universidad Juárez Autónoma de Tabasco.*

In this paper we use a model of three ordinary differential equations associated with a three Josephson Junctions Array Memory (JJAM) in order to optimally transition the system between two equilibrium states. For the computational solution of the optimal control problem we consider a conjugate gradient algorithm in a Hilbert space. We show the application of this methodology to transition between two stable equilibrium configurations; this kind of transition is used to define *Read/Write* memory operators. Also we show results associated with the transition from a stable equilibrium to an unstable equilibrium.

9. Inverse problems for wave propagation in heterogeneous media

Alberto Mercado Saucedo. *DMCC, Universidad Técnica Federico Santa María.*

We present an inverse problem regarding the recovering of coefficients in some wave equations with discontinuous main coefficient. These equations apear as a model of wave propagation in heterogeneous media. We will show how the stability of this inverse problem is proved, when the interface satisfies a given geometric hypothesis. The main tool is the use of adequate Carleman estimates. Also, we will present some related problems regarding models for wave propagation in beams and ongoing works regarding generalizations of the stated inverse problems.

10. On inverse source problems and controllability for the Stokes and Navier–Stokes equations.

Cristhian Montoya. *IM, Universidad Nacional Autónoma de México.*

In this talk, we will deal with recent works on inverse source problems and controllability for the Stokes and Navier–Stokes systems. More precisely, on inverse problems for the Stokes system with homogeneous Dirichlet boundary conditions, we will show a reconstruction formula for the external source from local and missing velocity measurements. The main tools are: a result on null controllability for the N–dimensional Stokes system with $N-1$ scalar controls, spectral analysis of the Stokes operator and Volterra equations. We also will present some numerical experiments of our formula. This work has been done in collaborations with G. C. García and A. Osses. Additionally, on controllability, we will present the local null controllability for the Navier–Stokes system with nonlinear Navier–slip conditions, where the internal controls have one vanishing component. In this case, the proof is based on Carleman estimates and fixed point arguments. This second result was done in collaboration with S. Guerrero.

11. Inverse electroencephalographic problem for bioelectrical sources and pathologies.

José Jacobo Oliveros Oliveros. *FCFM, Benemérita Universidad Autónoma de Puebla*.
Co-authors: María Montserrat Morín Castillo (FCE-BUAP), Andrés Fraguela Collar (FCFM-BUAP), L. Héctor Juárez Valencia (DM-DCBI, UAM-Izt), José Julio Conde Mones (DM-DCBI, UAM-Izt), Gregorio Garcia Aguilar (FP-BUAP), Emmanuel Estrada Aguayo (FCE-BUAP), Claudia Netzahualcoyotl Bautista (FCFM-BUAP), Héctor Ramírez Díaz (ITSEldorado).

The method of the electroencephalography is the best known among the non-invasive methods of the brain investigation, and it is based on the recording by electrodes located on the scalp of its electrical activity. This recording is known as Electroencephalogram (EEG). The electrical activity is generated by biolectrical sources of the brain and that reflect the dynamics of big neuron populations which have the capacity to work in synchrony. In this talk we consider the inverse problems of recovering from the EEG, bioelectrical sources defined in the brain (in the volume and cerebral cortex). We consider separately the case of dipolar (associated to epileptic focus) and distributed sources. In this cases these inverse problems are ill posed and regularization methods are applied in order to handle the numerical instability. Numerical examples are developed for simple and complex geometries using Fourier series and finite element method. For the case of epileptic sources on the cerebral cortex we propose mathematical models in order generate the EEG produced by that source. In the case of pathological situations associated to edemas, tumors and calcifications, we consider two models for the forward problem in order to generate the abnormal EEG produced by these pathologies. The uniqueness of the solution in guaranteed under some conditions. One boundary value problem is used to stablish relationships between EEG and bioelectrical sources and pathologies. Perspectives of the problem are presented.

Courses

1. Computational algorithms in inverse problems and applications.

Uri M. Ascher. *CSD, University of British Columbia*.

Much of the recent advances in the general area of applied inverse problems concerns computational approaches and corresponding algorithms. When PDE-based constraints are combined with often non-convex optimization of potentially ill-posed problems, the resulting algorithms and their implementation are rarely straightforward and are usually rather time consuming to work with.

I will discuss several computational approaches including optimization methods, randomized algorithms and data manipulation. Several applications will be employed to demonstrate the issues that arise and their resolution.

2. Numerical analysis of control problems for parabolic equations and systems.

Frank Boyer. *Université Paul Sabatier - Toulouse 3, Institut Universitaire de France*.

In this series of lectures, I will discuss some controllability problems for parabolic PDEs, in particular from the point of view of numerical analysis. Among the topics that will be addressed

- Penalized HUM approach.
- Relaxed observability inequalities.
- Discrete Carleman estimates.
- Discrete spectral properties of elliptic operators.

3. Introduction to the numerical solution of control and inverse problems.

L. Héctor Juárez Valencia. *DM-DCBI, Universidad Autónoma Metropolitana - Iztapalapa.*

The purpose of these lectures is to give a basic introduction to the numerical solution of optimal control problems and some inverse problems, mainly problems modeled by elliptic and parabolic PDE. In order to motivate some ideas in a simple framework, we will start first with the study and numerical solution of underdetermined and overdetermined finite dimensional algebraic linear systems. Once we get some experience with the finite dimensional case, we will employ the main ideas to study and solve control problems and inverse problems modeled by PDE. This traditional approach includes least squares techniques, penalization, regularization, optimality conditions, adjoint systems, saddlepoint problems, conjugate gradient and calculus of variations, among other.

Posters

1. Numerical solutions of boundary value problems in the plane for the electrical impedance equation: A pseudoanalytic approach.

Ariana G. Bucio Ramírez. *UPIITA, Instituto Politécnico Nacional*.
Co-authors: Raúl Castillo Pérez (ESIME-ZAC, IPN) and Marco Ramírez Tachiquín.

This work proposes a full numerical method and algorithm, for numerically approaching solutions for the Vekua equation, and explains the application of these solutions for studying the forward Dirichlet boundary value problem corresponding to the two-dimensional electrical impedance equation, when the conductivity is an arbitrary function fully defined within a bounded domain. The forward Dirichlet boundary value problem is solved bias this numerical method, based upon Pseudoanalytic Function Theory that does not requires additional regularization techniques for non-smooth domains.

The study of the solutions of the forward Dirichlet boundary value problem in the plane, corresponding to the Electrical Impedance equation

$$\text{div}\,(\sigma \text{ grad } u) = 0, \tag{2.2}$$

where $\sigma = \sigma_1(x)\sigma_2(y)$ is the conductivity function, and u is the electric potential, constitutes the base for analyzing the inverse problem, commonly known as Electrical Impedance Tomography.

The discovery of the relation between (2.2) in the plane, and the Vekua equation [13], by V. Kravchenko [7], and shortly after by K. Astala and L. Päivärinta [1], opened a complete new path for constructing numerical solutions of the forward problem corresponding to (2.2), based upon the modern Pseudoanalytic Function Theory [2]. This work is fully dedicated to emphasize a special property belonging to a novel numerical method [3][8], that achieves to approach solutions of the forward boundary value problem of (2.2), when considering a certain class of bounded domains with non-smooth boundaries, without employing additional regularization methods.

The reader can appreciate that, even there is not a criteria fr adequately identify the class of non-smooth boundaries that can be analyzed using this method, it is possible to infer that it will be valid for a wide variety of examples strongly related with physical cases.

The relevance of efficiently solving the forward problem for (2.2), if we are to solve the Electrical Impedance Tomography problem (also called inverse problem), was widely exposed in a variety of works, among which [12] is one of the most important. In this sense, the results posed in [7], and subsequently rediscovered in [1], are indeed very significant, because they allowed to sift out the rink for approaching the general solution of the Impedance Equation in the plane.

References

[1] K. Astala, L. Päivärinta (2006), *Calderon's inverse conductivity problem in the plane*, Annals of Mathematics, Vol. 163, pp. 265-299.

[2] L. Bers (1953), *Theory of Pseudoanalytic Functions*, IMM, New York University.

[3] A. Bucio R., R. Castillo-Perez, M.P. Ramirez T., *On the Numerical Construction of Formal Powers and their Application to the Electrical Impedance Equation*, 8th International Conference on Electrical Engineering, Computing Science and Automatic Control 2011, IEEE Catalog Number: CFP11827- ART,ISBN: 978-1-4577-1013-1.

[4] A. Bucio R., R. Castillo-Perez, M.P. Ramirez T.,C.M.A. Robles G., *A Simplified Method for Numerically Solving the Impedance Equation in the Plane*, 9th International Conference on Electrical Engineering, Computing Science and Automatic Control 2012, IEEE Catalog Number: CFP12827-CDR, ISBN: 978-1-4673-2168-6.

[5] R. Castillo-Perez., V. Kravchenko, R. Resendiz V. (2011), *Solution of boundary value and eigenvalue problems for second order elliptic operators in the plane using pseudoanalytic formal powers*, Mathematical Methods in the Applied Sciences, Vol. 34, Issue 4.

[6] V. V. Kravchenko (2009), *Applied Pseudoanalytic Function Theory*, Series: Frontiers in Mathematics, ISBN: 978-3-0346-0003-3.

[7] V. V. Kravchenko (2005), *On the relation of pseudoanalytic function theory to the two-dimensional stationary Schrödinger equation and Taylor series in formal powers for its solutions*, Journal of Physics A: Mathematical and General, Vol. 38, No. 18, pp. 3947-3964.

[8] C. M. A. Robles G., A. Bucio R., M. P. Ramirez T. (2012), *An Optimized Numerical Method for Solving the Two-Dimensional Impedance Equation*, Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2012, WCECS 2012, 24-26 October, 2012, San Francisco, USA, pp. 116-121.

[9] C. M. A. Robles G., A. Bucio R. and M. P. Ramirez T., *New Characterization of an Improved Numerical Method for Solving the Electrical Impedance Equation in the Plane: An Approach from the Modern Pseudoanalytic Function Theory*, IAENG International Journal of Applied Mathematics, ISSN: 1992-9986 (online version); 1992-9978 (print version), Vol. 43, Issue 1, pp.17-29, 2013.

[10] C. M. A Robles G., A. Bucio R., M. P. Ramirez T., (2012), *First characterization of a new method for numerically solving the Dirichlet problem of the two-dimensional Electrical Impedance Equation*,Proceedings of the World Congress on Engineering and Computer Science 2012 Vol I, WCECS 2012, ISBN: 978-988-19251-6-9, ISSN: 2078-0966.

[11] M. Robles-Gonzalez, V. Ponomaryov, M. Ramirez-Tachiquin, A. Bucio-Ramirez , *Numerical algorithm for electrical impedance tomography problem using pseudoanalytic approach* , The eigth international Kharkov symposium on physics and engineering of microwaves, millimeter and submilimeter waves (MSMW13) and workshop on terahertz technology (TERATECH13), Kharkiv, Ukraine, 2013.

[12] J.G. Webster (1990), *"Electrical Impedance Tomography"*,Adam Hilger, New York.

[13] I. N. Vekua (1962), *Generalized Analytic Functions,* International Series of Monographs on Pure and Applied Mathematics, Pergamon Press.

2. Modelos de medio conductor para generar EEG asociado a patologías en el cerebro.

Emmanuel Roberto Estrada Aguayo. *FCFM, Benemérita Universidad Autónoma de Puebla.*
Co-authors: José Jacobo Oliveros Oliveros (FCFM-BUAP), Andrés Fraguela Collar (FCFM-BUAP), María Montserrat Morín Castillo (FCE-BUAP), Gregorio García Aguilar (FP-BUAP) and Héctor Ramírez Díaz (ITSEldorado).

La Electroencefalografía es una de las técnicas más conocidas de investigación no invasiva del cerebro. Por medio de esta técnica se han detectado posibles anomalías en el cerebro ya que la conductividad eléctrica varía con diferentes situaciones patológicas tales como tumores, edemas y calcificaciones. El problema de determinar las anomalías a través del EEG es llamado Problema Inverso Electroencefalográfico y cae dentro de la categoría de los problemas mal planteados. Esto es debido a que existen diferentes configuraciones que pueden producir el mismo EEG y a que pequeñas variaciones en los datos de entrada pueden producir variaciones sustanciales en la localización de la fuente. La conductividad eléctrica de lesiones cerebrales varía con la situación patológica. En el caso de tumores. Se sabe que estos son silencio eléctrico, es decir, en la zona afectada no se refleja actividad eléctrica; sin embargo, una corriente eléctrica secundaria puede generarse alrededor del tumor. Para el caso de las calcificaciones se está considerando que dicha patología tiene una conductividad mucho menor comparada con el resto del cerebro sano por lo tanto se considera un dieléctrico. En este trabajo se propone un modelo matemático para reproducir el EEG asociado a una calcificación para lo cual se utiliza un modelo de medio conductor. La anomalía en el centro del cerebro estudiada representa a la glándula pineal calcificada lo cual se sabe que ocurre a una edad determinada en la primera década de vida. La glándula pineal humana crece en tamaño hasta el primer o segundo año de edad, permaneciendo estable después de ese periodo, aunque su peso se incrementa gradualmente a partir de la pubertad. Con este avance se pretende llegar a un algoritmo de localización espacial en el cerebro de la calcificación pineal usando como dato de entrada el EEG asociado.

3. An optimal regularization estimation for the Cauchy problem for the Laplace equation in a cylindrical domain.

Eduardo Hernández Montero. *FCFM, Benemérita Universidad Autónoma de Puebla.*
Co-authors: Andrés Fraguela Collar (FCFM-BUAP).

A. Fraguela and coworkers have recently working on develop a new methodology to solve inverse problems, broadly consisting on getting full description of theoretical output data in the equation $Ax = y$ (*admissible data*), and using *a priori* information to get proper projections $\mathcal{P}$ over this admissible data set such that $A^{-1}\mathcal{P}$ is continuous, where A is a compact operator having dense domain in a Hilbert space.

We show how this methodology provide an optimal regularization strategy on the solution of the Cauchy problem with data given on a part of the boundary in the framework of Laplace equation in a cylindrical domain, same boundary data completion problem as in [1] Cauchy data (Neumann and Dirichlet boundary conditions) is given on the Γ_0 basis of the region while no condition are given on the other basis Γ_a. The Dirichlet boundary condition is assumed zero in the side of the cylinder Σ.

In this case an admissible data is a couple (φ, ψ) in $H^{1/2}(\Gamma_0) \times L^2(\Gamma_0)$ for which the weak solution of the described Cauchy problem exists. A compact injection norm is defined in the admissible data set. By *a priori* information, the exact Cauchy data $(\varphi^\dagger, \psi^\dagger)$ associated with the perturbed one $(\tilde{\varphi}, \tilde{\psi})$ belongs to $\mathcal{M}_0$, a bounded set in the admissible data norm. Hence, the regularized solution is the trace over Γ_a of the weak solution to the Cauchy problem with data (φ^*, ψ^*), where (φ^*, ψ^*) is the element in $\mathcal{M}_0$ which minimize the distance to $(\tilde{\varphi}, \tilde{\psi})$ in the $L^2(\Gamma_0) \times L^2(\Gamma_0)$ norm.

The obtained regularization strategy is optimal; however, it depends on *a priori* information that is hardly ever known in real applications.

References

[1] A. B. Abda, J. Henry and F. Jday. Boundary data completion: the method of boundary value problem factorization. *IOP Publising Inverse Problems*, (27), 2011.

4. Robust Stackelberg controllability for parabolic equations.

Victor Hernández Santamaría. *DCA, Cinvestav.*

The aim of this poster is to present a Stackelberg strategy to control parabolic equations. We have one control, the leader, that is responsible for a null controllability property; additionally, we have a control the follower that solves a robust control objective. That means, that we seek for a saddle point of a cost functional. In this way, the follower control is insensitive to a broad class of external disturbances.

5. Problema bidimensional de identificación de la frontera de contacto entre un medio conductor y un conductor ideal.

Tishbe Pilarh Herrera Ramírez. *FCFM, Benemérita Universidad Autónoma de Puebla.*
Co-authors: Andrés Fraguela Collar (FCFM-BUAP).

El siguiente problema surge en las ciencias aplicadas. Para una región conductora eléctrica, utilizando métodos no invasivos, se desea identificar características desconocidas en su interior como un conductores ideales o un aislantes ideales. La modelación matemática permite formular esta problematica en el marco de la teoría de los problemas inversos.

Este trabajo da las bases para resolver problema de identificar la frontera de una inclusión interior conductora ideal. Para ello supondremos que para una semibanda conductora, existe una altura a partir de la cual la baanda se comporta como un conductor ideal. El problema de identificación a resolver es determinar dicha altura a partir de mediciones de potencial y corriente en la base de la misma.

Debido a que el problema implica un mal planteamiento, se propone una estrategia de regularización para obtener la altura en la banda a partir ciertas características sobre las mediciones en la base. Dichas características en las mediciones son determinadas por el comportamiento de una región en la cual se sabe existe este cambio de conductividad.

6. On the controllability of diffusion processen on a sphere in $\mathbb{R}^3$.

Diana Assaely León Velasco. *DM-DCBI, Universidad Autónoma Metropolitana - Iztapalapa*.

Many physico–chemical phenomena modeled by partial differential equations take place on the surface of planet Earth; it makes sense therefore to attempt controlling some of them (pollution for example). This evidence leads naturally to control and controllability problems for surfaces of $\mathbb{R}^3$, spheres in particular. Looking at the literature shows that, for example, the control of diffusion processes on surfaces of $\mathbb{R}^3$ has not attracted much attention, yet, despite the fact that such problems have potentially many applications. Then, the goal of this work is to study computationally the controllability of a diffusion process on the surface of a sphere in $\mathbb{R}^3$. To achieve this goal, we employ a methodology combining finite differences for the time discretization, finite elements for the space approximation, and a conjugate gradient algorithm for the iterative solution of the discrete control problems.

7. Improved characterization of naturally fractured-vuggy carbonate reservoirs through a statistical multivariate analysis.

Carlos Minutti. *IIMAS, Universidad Nacional Autónoma de México*.
Co-authors: Susana Gómez (IIMAS,UNAM), Gustavo Ramos (IIMAS,UNAM), Rodolfo Camacho (PEMEX), Mario Vasquez (PEMEX).

Characterization of naturally fractured carbonate reservoirs has been a major challenge in the oil, gas and underground water industry. The world's largest oil and gas reserves are present in these giant carbonate reservoirs. Dynamic reservoir modeling is still under study, specially due to the triple porosity which adds another dimension of complexity to the characterization of these reservoirs, with multiple solutions [1], [2], [3].

In order to characterize these reservoirs through well test analysis, we use a triple-porosity double-permeability model, and a triple-porosity single-permeability fractal model. Using the analytical solution of the model, a statistical sensitivity analysis of a large number of synthetic cases is performed, and the behavior of the different parameters involved in the model is studied by averaging the effect of each parameter in the pressure and the magnitude of this effect. Also the importance of each parameter over different time values is evaluated. By unifying this information, an ad-hoc methodology for reservoir characterization (with the studied model) is proposed.

This methodology significantly reduces the estimation error of the reservoir parameters, from 29% to 11% on the median overall error for the less-sensitivity parameters, making it possible to estimate these parameters to changes in wellbore pressure. It also predicts the range of values in which each parameter has a higher estimation error and can eliminate multiple solutions that can be reached with an optimization algorithm due to numerical precision.

We present results on Mexican carbonate reservoirs and make a comparison with results using commercial dual porosity software. We show that our characterization increases the amount of information of the reservoir, gets more accurate fit, and demonstrate that considering triple porosity is crucial for these types of reservoirs.

References

[1] R. Camacho-V, S. Gmez, M. Vsquez-C, N. Fuenleal-M, T. Castillo-R, G. Ramos, C. Minutti-M, A. Mesejo, G. Fuentes-C, *Well testing characterization of heavy-oil naturally fractured vuggy reservoirs*, In Society of Petroleum Engineering, 2014.

[2] S. Gomez, G. Ramos, A. Mesejo, R. Camacho, M. Vsquez and N. del Castillo, *Well test analysis of naturally fractured vuggy reservoirs with an analytical triple porosity - double permeability model and a global optimization method*, In Oil Gas Sci. Technol. - Rev. IFP Energies nouvelles, 2014.

[3] S. Gomez, R. Camacho, M. Vasquez, G. Ramos, N. del Castillo and J.A. Mesejo, *Well test characterization of naturally fractured vuggy reservoirs, with a global optimization method*, In Society of Petroleum Engineering, OTC-24762-MS. 2014.

[4] Rodolfo Camacho-V., Susana Gmez, Mario Vsquez-C, Norma Fuenleal-M, Toms Castillo-R, Gustavo Ramos, Carlos Minutti M, Alejandro Mesejo, Gorgonio Fuentes-C. *Well Testing Characterization of Heavy-Oil Naturally Fractured Vuggy Reservoirs*. SPE 171078. Pg. 1-13. 2014.

[5] C. Minutti, G. Ramos and S. Gomez, *Robust Characterization of Naturally Fractured Carbonate Reservoirs through Sensitivity Analysis and Noise Propagation reduction*. SPE -181182-MS 2016.

8. Problema directo de identificación de fuentes en la corteza cerebral de tipo dipolar asociadas a focos epilépticos.

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La electroencefalografía ha sido usada ampliamente para el estudio de la epilepsia. Para el caso de focos epilépticos que se localizan en el volumen cerebral, se han usado modelos matemáticos que reproducen satisfactoriamente el Electroencefalograma (EEG) en el cuero cabelludo generado por esos focos. Esto ha permitido el estudio del problema de identificación de estos focos. Sin embargo, esto no ocurre para focos epilépticos ubicados en la corteza cerebral, es decir dichos modelos no han podido reproducir satisfactoriamente el EEG. En este trabajo se proponen tres modelos matemáticos para estudiar este problema. Con ello, se pueden establecer correlaciones entre el EEG sobre el cuero cabelludo y las fuentes (focos epilépticos) en la corteza cerebral. Se analizan casos en los que puede estar ubicada la fuente sobre corteza a saber: en el volumen de la corteza cerebral, superficie de la corteza cerebral y en la combinación de ambos.

9. Calderon's inverse problem for circular inclusions.

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We will study the solution of the Calderons Inverse Problem. It will be assumed that the conductivity is piecewise constant and takes only two distinct positive values, taking a constant value in a circular inclusion D contained in the interior of a circular region Ω and another constant value in the complement of D. First we construct the Neumann-to-Dirichlet operator in a explicit way and then we solve the inverse problem using this operator in a optimization problem looking for the parameters that characterize the circular inclusion; center and radius.

In the methodology that we are developing we use the Neumann-to-Dirichlet operator in order to obtain a relation between the Cauchy data in terms of the parameters of the inclusion. This relation characterizes the Cauchy data if there is no errors in the measures. We call to this type of data: "admissible data". So the admissible data are the data that could be reproduced with a circular inclusion. Then we use a subset of admissible data with convenient characteristics, using a priori information, in order that the inverse problem for data measured with error be a well posed problem. In this case we take a subset of admissible data such that the data has only a finite number of Fourier coeficients. Finally, we solve the inverse problem for data measured with error using the convenient subset of admissible data in a minimum distance problem.