

MATH 319 - SEC 003, SPRING 2014. HOMEWORK 1

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Due : Monday, February 3rd.

Please show all your work and/or justify your answers.

Problem: Verify by substitution that each given function is a solution of the given differential equation:

$$x^2 y'' + 5xy' + 4y = 0, y_1 = \frac{1}{x^2}, y_2 = \frac{\ln x}{x^2}.$$

Problem Use your knowledge of derivatives to find one solution of

$$x^3 y' + y^2 = -x^4$$

Section 1.1 Problem 1: Consider the ordinary differential equation

$$y' = 3 - 2y$$

Draw a direction field and determine the behavior of the solution as $t \rightarrow \infty$.

Section 1.1 Problem 22: A spherical raindrop evaporates at a rate proportional to its surface area. Write a differential equation for the volume of the raindrop as a function of time.

Section 1.1 Problem 23: Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object itself and the temperature of its surroundings. Suppose that the ambient temperature is 70°F and that the rate constant is 0.05min^{-1} . Write a differential equation for the temperature of the object at any time.

Problem 1 Section 1.2 Solve each of the following initial value problems and plot the solutions for several values of y_0 . Then describe in a few words how the solutions resemble, and differ from, each other

- (a) $dy/dt = -y + 5, y(0) = y_0$
- (b) $dy/dt = -2y + 5, y(0) = y_0$
- (c) $dy/dt = -2y + 10, y(0) = y_0$.

Problem 9 Section 1.2 The falling object in Example 2 satisfies the initial value problem

$$\frac{dv}{dt} = 9.8 - \frac{v}{5}, v(0) = 0.$$

- (a) Find the time that must elapse for the object to reach 98% of its limiting velocity
- (b) How far does the object fall in the time found in part (a)?

Problem 10 Section 1.2 Modify Example 2 so that the falling object experiences no air resistance

- (a) Write does the modified initial value problem
- (b) Determine how long it takes for the object to reach ground
- (c) Determine its velocity at the time of impact

Problem 19 Section 1.2 Your swimming pool containing 60,000 gal of water has been contaminated by 5 kg of nontoxic dye that leaves a swimmer's skin an unattractive green. The pool's filtering system can take water from the pool, remove the dye, and return the water to the pool at a rate of 200 gal/min.

- (a) Write down the initial value problem for the filtering process; let $q(t)$ be the amount of dye in the pool at time t
- (b) Solve the problem in part (a)
- (c) You have invited several dozen friends to a pool party that is scheduled to begin in 4 h. You have also determined that the effect of the dye is imperceptible if its concentration is less than 0.02 g/gal. Is your filtering system capable of reducing the dye concentration to this level within 4h?
- (d) Find the time T at which the concentration of dye first reaches the value of 0.02 g/gal.
- (e) Find the flow rate that is sufficient to achieve the concentration 0.02 g/gal within 4 h.