MATH 319 - SEC 003, SPRING 2014. HOMEWORK 10

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Due : Friday, April 18.

Please show all your work and/or justify your answers.

Section 5.3 Problems 7 and 8 Determine a lower bound for the radius of convergence of series solutions about each given point x_0 for the given differential equation

7. $(1+x^3)y'' + 4xy' + y$; $x_0 = 0, x_0 = 2$

8. $xy'' + y = 0$; $x_0 = 1$

Section 5.3 Problem 10 (The Chebyshev Equation) The Chevychev differential equation is

$$(1-x^2)y'' - xy' + \alpha^2 y = 0,$$

where α is a constant.

(a) Determine two solutions in powers of x for $|x| < 1$ and show that they form a fundamental set of solutions

(b) Show that if α is a nonnegative integer n, then there is a polynomial solution of degree n. These polynomials, when properly normalized, are called the Chevyshev polynomials. They are very useful in problems that require a polynomial approximation to a function defined on $-1 \leq x \leq 1$.

(c) Find a polynomial solution for each of the cases $\alpha = n = 0, 1, 2, 3$.

Section 5.3 Problems 13 and 14 Find the first non-zero terms in each of two power series solutions about the orginin. Show that they form a fundamental set of solutions. What do you expect the radius of convergence to be for each solution?

13. $(\cos x)y'' + xy' - 2y = 0$

14. $e^{-x}y'' + \ln(1+x)y' - xy = 0$

Section 5.4 Problems 9 and 10 Determine the general solution of the given differential equation that is valid in any interval not including the singular point

9. $x^2y'' - 5xy' + 9y = 0$

10. $(x-2)^2y'' + 5(x-2)y' + 8y = 0$

Section 5.4 Problems 31-33 Find all singular points of the given equation and determine whether each one is regular or irregular

31. $x^2y'' - 3(\sin x)y' + (1+x^2)y = 0$

32. $xy'' + y' + (\cot x)y = 0$

33. $(\sin x)y'' + xy' + 4y = 0$

Section 5.4 Problem 36 Find all values of β for which all solutions of $x^2y'' + \beta y = 0$ approach zero as $x \to 0$.