

MATH 319 - SEC 003, SPRING 2014. HOMEWORK 12

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Due : Friday, May 2nd.

Please show all your work and/or justify your answers.

Section 5.6 Problems 1 and 3

- (a) Find all the regular singular points of the given differential equation
(b) Determine the indicial equation and the exponent of the singularity for each regular singular point.

1.- $xy'' + 2xy' + 6e^x y = 0$

3.- $x(x-1)y'' + 6x^2y' + 3y = 0$.

Section 5.6 Problems 14 and 15

- (a) Show that $x = 0$ is a regular singular point of the given differential equation
(b) Find the exponents at the singular point $x = 0$
(c) Find the first three nonzero terms in each of the two solutions (not multiples of each other) about $x = 0$.

14.- $xy'' + 2xy' + 6e^x y = 0$

15.- $x(x-1)y'' + 6x^2y' + 3y = 0$.

Section 6.1 Problems 18 and 19 Use integration by parts to find the Laplace transform of the following functions. Here n is a positive integer and a is a real constant.

18.- $f(t) = t^n e^{at}$

19.- $f(t) = t^2 \sin(at)$

Section 6.2 Problem 21 Use the Laplace transform to solve the given initial value problem

$$y'' - 2y' + 2y = \cos t, y(0) = 1, y'(0) = 0.$$

Section 6.2 Problem 24 Find the Laplace transform $Y(s) = \mathcal{L}[y]$ of the solution of the given initial value problem. A method of determining the inverse transform is developed in Section 6.3

$$y'' + 4y = \begin{cases} 1, & 0 \leq t \leq \pi, \\ 0, & \pi \leq t < \infty; \end{cases} \quad y(0) = 1, y'(0) = 0.$$

Section 6.4 Problems 1 and 2 In each of the following problems

- (a) Find the solution of the initial value problem
(b) Draw the graph of the solution and the forcing function; explain how they are related

1.- $y'' + y = f(t); y(0) = 0, y'(0) = 1; f(t) = \begin{cases} 1, & 0 \leq t < 3\pi \\ 0, & 3\pi \leq t < \infty \end{cases}$

2.- $y'' + 2y' + 2y = h(t); y(0) = 0, y'(0) = 1; h(t) = \begin{cases} 1, & \pi \leq t < 2\pi \\ 0, & 0 \leq t < \pi \text{ and } t \geq 2\pi \end{cases}$