MATH 319 - SEC 003, SPRING 2014. HOMEWORK 13

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Due : Friday, May 9.

Please show all your work and/or justify your answers.

Problem. Determine the equilibrium temperature distribution for a one-dimensional rod with constant thermal properties with the following sources and boundary conditions:

(a) $Q = 0,\ u(0) = 0,\ u(L) = T$

(b) $Q = 0,\ u(0) = T,\ \frac{\partial u}{\partial x}(L) = \alpha$

Note: Here, equilibrium temperature distributions are solutions independent of time.

Section 10.5 Problem 8. Find the solution of the heat conduction problem

$$u_t = \frac{1}{4}u_{xx}, 0 < x < 2, t > 0;$$
$$u(0,t) = 0, u(2,t) = 0, t > 0;$$
$$u(x,0) = 2\sin(\pi x/2) - \sin \pi x + 4 \sin 2\pi x, 0 < x < 2.$$

Consider the conduction of heat in a rod 40 cm in length whose ends are maintained at 0°C for all $t > 0$. In each of Problems 9 and 10 find the expression of the temperature $u(x,t)$ if the initial temperature distribution in the rod is the given function.

9.-

$$u(x,0) = 50^\circ\text{C}, 0 < x < 40$$

10.-

$$u(x,0) = \begin{cases} x, & 0 \leq x < 20 \\ 40 - x, & 20 \leq x \leq 40. \end{cases}$$

Problem : Numerically compute solutions to the heat equation

$$\begin{aligned} &u_t = ku_{xx}, 0 \leq x \leq 1,\ k = 1 \\ &u(0,t) = u(1,t) = 0 \\ &u(x,0) = f(x) = \begin{cases} 0, & 0 \leq x < 0.5 \\ x - 0.5, & 0.5 \leq x < 0.75 \\ 1 - x, & 0.75 < x < 1 \end{cases} \end{aligned}$$

using the numerical scheme

$$u_j^{(m+1)} = u_j^{(m)} + s\left(u_{j-1}^{(m)} - 2u_j^{(m)} + u_{j+1}^{(m)}\right), j = 1, \ldots, N-1,\ s = k\Delta t/\Delta x^2$$

with $\Delta x = 0.1$ ($N = 10$). Do for various s (discuss stability):

$$s = 0.49,\ s = 0.50,\ s = 0.51,\ s = 0.52$$

Note: If you create your own code, include it in your homework. If you don't have access to matlab or any other compiler, do it by hand and report the numerical results in a table (3 time steps).