MATH 319 - SEC 003, SPRING 2014. HOMEWORK 2

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Due : Monday, February 10.

Please show all your work and/or justify your answers.

Problem: Verify that if c is a constant, then the function defined piecewise by

$$y(x) = \begin{cases} 0 & \text{for } x \leq c \\ (x-c)^2 & \text{for } x > c \end{cases}$$

satisfies the differential equation $y' = 2\sqrt{y}$ for all x.

Problem: Construct a figure illustrating the fact that the initial value problem

$$\begin{aligned} y' &= 2\sqrt{y} \\ y(0) &= 0 \end{aligned}$$

has infinitely many solutions.

Section 2.1 Problems 13-20 In each of the problems 13 through 20 find the solution of the given initial value problem.

- **13.** $y' - y = 2te^{2t}, y(0) = 1$
- **14.** $y' + 2y = te^{-2t}, y(1) = 0$
- **15.** $ty' + 2y = t^2 - t + 1, y(1) = 1/2, t > 0$
- **16.** $y' + (2/t)y = \cos(t)/t^2, y(\pi) = 0, t > 0$
- **17.** $y' - 2y = e^{2t}, y(0) = 2$
- **18.** $ty' + 2y = \sin(t), y(\pi/2) = 1, t > 0$
- **19.** $t^3y' + 4t^2y = e^{-t}, y(-1) = 0, t < 0$
- **20.** $ty' + (t+1)y = t, y(\ln(2)) = 1, t > 0$

Section 2.1 Problem 29: Consider the initial value problem

$$y' + \frac{1}{4}y = 3 + 2\cos(2t), y(0) = 0.$$

(a) Find the solution of this initial value problem and describe its behavior for large t.

(b) Determine the value of t for which the solution first intersects the line $y = 12$.

Section 2.1 Problem 30:

Find the value of y_0 for which the solution of the initial value problem

$$y' - y = 1 + 3\sin(t), y(0) = y_0.$$

remains finite as $t \to \infty$.