MATH 319 - SEC 003, SPRING 2014. HOMEWORK 3

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Due : Monday, February 17.

Please show all your work and/or justify your answers.

Section 2.2 Problems 1,2,5,6: In each of the following problems solve the given differential equation

1. $y' = x^2/y$
2. $y' = \frac{x^2}{y(1+x^3)}$
5. $y' = (\cos^2 x)(\cos^2 2y)$
6. $xy' = (1-y^2)^{1/2}$

Section 2.2 Problem 21 Solve the initial value problem

$$y' = (1+3x^2)/(3y^2-6y), \ y(0) = 1.$$

and determine the interval in which the solution is valid.

Hint: To find the interval of definition, look for points where the integral curve has a vertical tangent.

Section 2.2 Problem 30: Solve

$$\frac{dy}{dx} = \frac{y-4x}{x-y}$$

by transforming this homogeneous equation into a separable equation as seen in class. Read page 49 for details. Complete parts (a)-(f) in pages 49 and 50.

Section 2.3 Problem 3: A tank originally contains 100 gal of fresh water. Then water containing 1/2 lb of salt per gallon is poured into the tank at a rate of 2 gal/min, and the mixture is allowed to leave at the same rate. After 10 min the process is stopped, and fresh water is poured into the tank at a rate of 2 gal/min, with the mixture again leaving a t the same rate. Find the amount of salt in the tank at the end of the 10 min.

Section 2.3 Problem 29: Suppose that a rocket is launched straight up from the surface of earth with initial velocity $v_0 = \sqrt{2gR}$, where R is the radius of the earth. Neglect air resistance.

(a) Find an expression for the velocity v in terms of the distance x from the surface of the earth.
(b) Find the time required for the rocket to go 240,000 mi (the approximate distance from the earth to the moon). Assume $R = 4000$ mi.

Section 2.4 Problem 16 Solve the initial value problem

$$y' = \frac{t^2}{y(1+t^3)}, y(0) = y_0,$$

and determine how the interval on which the solution exists depends on the initial value y_0.

Section 2.5 Problem 15: Suppose that a certain population obeys the logistic equation $dy/dt = ry(1-y/K)$.

(a) If $y_0 = K/3$, find the time τ at which the initial population has doubled. Find the value of τ corresponding to $r = 0.025$ per year.

(b) If $y_0/K = \alpha$, find the time T at which $y(T)/K = \beta$, where $0 < \alpha, \beta < 1$. Observe that $T \to \infty$ as $\alpha \to 0$ or as $\beta \to 1$. Find the value of T for $r = 0.025$ per year, $\alpha = 0.1$, and $\beta = 0.9$.