

MATH 319 - SEC 003, SPRING 2014. HOMEWORK 6

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Due : Wednesday, March 12.

Please show all your work and/or justify your answers.

Section 3.3 Problem 23 Consider the initial value problem

$$3u'' - u' + 2u = 0, u(0) = 2, u'(0) = 0.$$

- (a) Find the solution $u(t)$ of this problem
- (b) For $t > 0$ find the first time at which $|u(t)| = 10$.

Section 3.3 Problem 26 Consider the initial value problem

$$y'' + 2ay' + (a^2 + 1)y = 0, y(0) = 1, y'(0) = 0.$$

- (a) Find the solution $y(t)$ of this problem
- (b) For $a = 1$ find the smallest T such that $|y(t)| < 0.1$ for $t > T$
- (c) Repeat part (b) for $a = 1/4, 1/2$ and 2 .
- (d) Using the results of parts (b) and (c), plot T versus a and describe the relation between T and a .

Section 3.3 Problem 27 Show that $W(e^{\lambda t} \cos \mu t, e^{\lambda t} \sin \mu t) = \mu e^{2\lambda t}$.

Section 3.3 Problem 29 Using Euler's formula, show that

$$\cos t = \frac{e^{it} + e^{-it}}{2}, \sin t = \frac{e^{it} - e^{-it}}{2i}.$$

Section 3.3 Problem 32 Let the real-valued functions p and q be continuous on the open interval I , and let $y = u(t) + iv(t)$ be a complex-valued solution of

$$(0.0.1) \quad y'' + p(t)y' + q(t)y = 0,$$

where u and v are real-valued functions. Show that u and v are also solutions of the equation above (0.0.1).

Hint: Substitute $y = u(t) + iv(t)$ in equation (0.0.1) and separate into real and imaginary parts.

Section 3.4 Problems 7-10 In each of the problems 7 through 10 find the general solution of the given differential equation.

7. $4y'' + 17y' + 4y = 0$

8. $16y'' + 24y' + 9y = 0$

9. $25y'' - 20y' + 4y = 0$

10. $2y'' + 2y' + y = 0$

Section 3.4 Problems 27-28 In each of the problems 27 and 28 use the method of reduction of order to find a second solution of the given differential equation

27. $xy'' - y' + 4x^3y = 0, x > 0; y_1(x) = \sin x^2$

28. $(x-1)y'' - xy' + y = 0, x > 1; y_1(x) = e^x$