Math 319: *Techniques in Ordinary Differential Equations* Spring 2014

Quiz 1

Time: 10 minutes

Name:______________________

1. Find the general solution of the following differential equation: $y' + 2y = 5e^{-t}$. Also describe how the solutions behave as $t \to \infty$. [6 Points]

2. The acceleration of a rocket travelling upward as a function of its height from the ground is given by $a(h) = 10 + h$ meter/sec^2. Find the *velocity* of the rocket when it is 100 meters above the ground. [4 Points]

① $\mu(t) = e^{\int 2dt} = e^{2t}$.

Mult. by e^{2t} to get

$$e^{2t}y' + 2e^{2t}y = 5e^{t}$$

$$\frac{d}{dt}(e^{2t}y) = 5e^{t}$$

$$e^{2t}y = 5e^{t} + C$$

$$\boxed{y = 5e^{-t} + Ce^{-2t}}$$

$\boxed{\text{As } t\to\infty,\ y(t) \to 0.}$

② $a(h) = 10 + h$, By chain rule, $a = \frac{dv}{dt} = \frac{dv}{dh}\cdot\frac{dh}{dt} = v\frac{dv}{dh}$

So $v\frac{dv}{dh} = 10 + h$. Let v_0 = velocity at height 100 meters.

At height 0, velocity is 0 since the rocket starts at rest.

$$\int_0^{v_0} v\,dv = \int_0^{100} 10 + h\,dh$$

$$\frac{v_0^2}{2} = 1000 + \frac{100^2}{2} = 6000$$

$$\boxed{v_0 = \sqrt{12000} \text{ meter/sec.}}$$