MATH 319 - SEC 003, SPRING 2014.
PRACTICE PROBLEMS FOR MIDTERM 2

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Problem Find the general solution to each of the following equations

- $y'' + 6y' + 13y = 0$
- $y'' + 2y' - 3y = 0$
- $4y'' - 4y' + y = 0$
- $4y'' + 9y = 0$

Problem Find the solution of the given initial value problem.

- $y'' + 4y' + 5y = 0,\ y(0) = 0, y'(0) = 2$
- $y'' + 2y' + 2y = 0,\ y(\pi/4) = 5, y'(\pi/4) = -1$
- $5u'' + 2u' + 7u = 0, u(0) = 2, u'(0) = 1$

Problem An equation of the form

$$t^2\frac{d^2y}{dt^2} + \alpha t\frac{dy}{dt} + \beta y = 0, t > 0,$$

where α and β are real constants, is called an Euler equation

(a) Let $x = \ln t$ and calculate dy/dt and d^2y/dt^2 in terms of dy/dx and d^2y/dx^2.

(b) Use the results of part (a) to transform the equation above into

$$\frac{d^2y}{dx^2} + (\alpha - 1)\frac{dy}{dx} + \beta y = 0.$$

Observe that the last equation has constant coefficients. If $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions of the last equation, $y_1(\ln t)$ and $y_2(\ln t)$ form a set of solutions of the former one.

(c) Apply the method here to solve the following equation

$$t^2y'' + 7ty' + 10y = 0, t > 0$$

Problem Solve each of the given initial value problem

- $9y'' + 6y' + 82y = 0, y(0) = -1, y'(0) = 2$
- $y'' - 6y' + 9y = 0, y(0) = 0, y'(0) = 2$

Problem Consider the initial value problem

$$9y'' + 12y' + 4y = 0, y(0) = a > 0, y'(0) = -1$$

(a) Solve the initial value problem

(b) Find the critical value of a that separates solutions that become negative from those that are always negative

Problem Use the method of reduction of order to find a second solution of the given differential equation

- $t^2y'' - 4ty' + 6y = 0, t > 0, y_1(t) = t^2$
- $t^2y'' + 3ty' + y = 0, t > 0, y_1(t) = t^{-1}$
- $x^2y'' + xy' + (x^2 - 0.25)y = 0, x > 0; y_1(x) = x^{-1/2}\sin x$

Problem Consider the differential equation

$$\frac{d^2y}{dx^2} - \frac{3}{x}\frac{dy}{dx} + \frac{4}{x^2}y = 0.$$

Find one solution of the form $y_1(x) = x^r$. The use the method of reduction of order to find a second solution $y_2(x)$ to combine with $y_1(x)$ to find the general solution.

Problem Find a general solution of the given differential equation

- $y'' - 2y' - 3y = 3e^{2t}$
- $y'' + 2y' = 3 + 4\sin 2t$
- $2y'' + 3y' + y = t^2 + 2\sin t$
- $y'' - 4y' - 12y = 3e^{5t} + \sin(2t) + te^{4t}$

Problem Find the solution of the given initial value problem

- $y'' + y' - 2y = 2t,\ y(0) = 0, y'(0) = 1$
- $y'' - 2y' + y = te^t + 4, y(0) = 1, y'(0) = 1$

Problem Use the method of variation of parameters to find a particular solution. Then check your answer using the method of undetermined coefficients

- $y'' - y' - 2y = 2e^{-t}$
- $y'' + 2y' + y = 3e^{-t}$

Problem Find the general solution of the given differential equation. Here $g(t)$ is an arbitrary continuous function.

- $y'' + 9y = 9\sec^2 3t, 0 < t < \pi/6$
- $y'' - 5y' + 6y = g(t)$
- $y'' + 4y = g(t)$
- $y'' - 2y' + y = \frac{e^t}{t^2+1}$
- $ty'' - (t+1)y' + y = t^2$

Problem Verify that y_1 and y_2 satisfy the corresponding homogeneous equation. Then find a particular solution of the non homogeneous equation.

- $t^2y'' - 2y = 3t^2 - 1, t > 0; y_1(t) = t^2, y_2(t) = t^{-1}$
- $ty'' - (1+t)y' + y = t^2e^{2t}, t > 0;\ y_1(t) = 1 + t, y_2(t) = e^t$
- $x^2y'' + xy' + (x^2 - 0.25)y = 3x^3\sin x, x > 0;\ y_1(x) = x^{-1/2}\sin x,\ y_2(x) = x^{-1/2}\cos x$

Problem Determine the radius of convergence of the given power series

- $\Sigma_{n=0}^{\infty} \frac{n}{2^n} x^n$
- $\Sigma_{n=1}^{\infty} \frac{(2x+1)^2}{n^2}$
- $\Sigma_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n}$

Problem Determine the Taylor series about the point x_0 for the given function. Also determine the radius of convergence of the series.

- $\sin x, x_0 = 0$
- $\ln x, x_0 = 1$

Problem In each of the following problems,

(a) Seek power series solutions of the given DE about the given point x_0; find the recurrence relation

(b) Find the first four terms in each of the two solutions y_1, y_2

(c) By evaluating the Wronskian, show that y_1 and y_2 form a fundamental set of solutions.

(d) If possible, find the general term in each solution

- $y'' - xy' - y = 0, x_0 = 0$
- $(2 + x^2)y'' - xy' + 4y = 0, x_0 = 0$
- $(1 + x^2)y'' - 4xy' + 6y = 0, x_0 = 0$
- $(4 - x^2)y'' + 2y = 0, x_0 = 0$
- $2y'' + (x + 1)y' + 3y = 0, x_0 = 2$

Problem Consider the initial value problem $y' = \sqrt{1 - y^2}, y(0) = 0$.

(a) Show that $y = \sin x$ is the solution of the initial value problem (locally)

(b) Look for a solution of the initial value problem in the form of a power series about $x = 0$. Find the coefficients up to the terms x^3 in this series.