Discrete Mathematics 59 (1986) 257–265
North-Holland

ON KERNEL-PERFECT CRITICAL DIGRAPHS

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Received 18 June 1984

In this paper we investigate new sufficient conditions for a digraph to be kernel-perfect (KP) and some structural properties of kernel-perfect critical (KPC) digraphs. In particular, it is proved that the asymmetrical part of any KPC digraph is strongly connected. A new method to construct KPC digraphs is developed. The existence of KP and KPC digraphs with arbitrarily large dichromatic number is also discussed.

1. Introduction

For general concepts we refer the reader to [1]. Let D be a digraph; $V(D)$ and $F(D)$ or FD will denote the sets of vertices and arcs of D respectively. Sometimes we write u_1u_2 instead of (u_1, u_2). If D_0 is a subdigraph (resp. induced subdigraph) of D we write $D_0 \subset D$ (resp. $D_0 \subset^* D$). If $S_1, S_2 \subset V(D)$, the arc u_1u_2 of D will be called an S_1S_2-arc whenever $u_1 \in S_1$ and $u_2 \in S_2$; $D[S_1]$ will denote the subdigraph of D induced by S_1 and $D[S_1, S_2]$ the subdigraph of D with vertex-set $S_1 \cup S_2$ and whose arcs are the S_1S_2-arcs of D. An arc $u_1u_2 \in F(D)$ is called *asymmetrical* (resp. *symmetrical*) if $u_2u_1 \notin F(D)$ (resp. $u_2u_1 \in F(D)$). The *asymmetrical part* of D (resp. *symmetrical part* of D), which is denoted by $\mathrm{Asym}(D)$ (resp. $\mathrm{sym}(D)$), is the spanning subdigraph of D whose arcs are the asymmetrical (resp. symmetrical) arcs of D; D is called an oriented graph if $\mathrm{Asym}(D) = D$. The directed cycle of length n is denoted by $\vec{C}_n$. The set $I \subset V(D)$ is *independent* if $\mathrm{FD}[I] = \emptyset$. A *kernel* N of D is an independent set of vertices such that for each $z \in V(D) - N$ there exists a zN-arc in D. A *semikernel* S of D is an independent set of vertices such that for every $z \in V(D) - S$, for which there exists an Sz-arc, there also exists a zS-arc.

A digraph D is called:

(i) *quasi R-digraph* if every proper induced subdigraph of D has a kernel;

(ii) *R-digraph* if every non empty induced subdigraph of D has a non empty semikernel;

(iii) *R^--digraph* if D is a quasi R-digraph and has no kernel.

The following result was proved by Neumann–Lara [12].

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Theorem 1.1. *D is an R-digraph if and only if every induced subdigraph of D has a kernel.*

Therefore a quasi R-digraph is either an R-digraph or an R^--digraph; R-digraphs (resp. R^--digraphs) are just kernel-perfect graphs (resp. kernel-perfect critical graphs) in the terminology of Duchet and Meyniel [4].

Sufficient conditions for the existence of kernels in a digraph have been investigated by several authors namely Von Neumann and Morgenstern [17], Richardson [14–16], Duchet and Meyniel [2–4] and Galeana-Sánchez and Neumann-Lara [7, 12]. In this paper we study new sufficient conditions for a digraph to be an R-digraph and structural properties of R^--digraphs. In particular it is proved that the asymmetrical part of an R^--digraph is strongly connected. A new method to construct R^--digraphs is developed.

Relations between quasi R-digraphs and its asymmetric part and the existence of R-digraphs with arbitrarily large dichromatic number are also investigated.

2. The asymmetrical part of quasi R-digraphs

Most of this section is based on Theorem 2.1.

Theorem 2.1. *Suppose that $V(D)$ has a partition $\{V_1, V_2\}$ such that every V_1V_2-arc in D is symmetric and $D[V_1]$ and $D[V_2]$ are R-digraphs. Then D is an R-digraph.*

Proof. Let D' be an induced subdigraph of D. If $D' \subset D[V_1]$ or $D' \subset D[V_2]$, D' has a kernel. In the opposite case, any kernel of $D' \cap D[V_1]$ is a semikernel of D'. Then D is an R-digraph. □

A corollary of Theorem 2.1 is

Theorem 2.2. *If D is an R^--digraph, there is no partition $\{V_1, V_2\}$ of $V(D)$ such that $D[V_1, V_2] \subset \mathrm{sym}(D)$; in other words, $\mathrm{Asym}(D)$ is strongly connected.*

As direct consequences of Theorem 2.2 we get

Corollary 2.1 (Duchet [2]). *Every R^--digraph is strongly connected.*

Corollary 2.2 (Duchet and Meyniel [4]). *If $\mathrm{Asym}(D)$ is acyclic, then D is an R-digraph.*

An important application of Corollary 2.2 is

Theorem 2.3. *If $\mathrm{Asym}(D) = \vec{C}_n$, then D is a quasi R-digraph and $D - f$ is an*

R-digraph for every $f \in F(\mathrm{Asym}(D))$.

Define the digraph $C = \vec{C}_n(j_1, j_2, \ldots, j_k)$ by

$$V(C) = \{0, 1, \ldots, n-1\},$$
$$F(C) = \{uv \mid v - u \equiv j_s \,(\mathrm{mod}\, n) \text{ for } s = 1, \ldots, k\}.$$

An application of Theorem 2.3 is

Theorem 2.4. *If* $2 \leqslant r \leqslant [\frac{1}{2}n]$, *then* $C = \vec{C}_n(1, \pm 2, \pm 3, \ldots, \pm r)$ *is an R-digraph or an* R^-*-digraph depending on whether* $n \equiv 0 \bmod (r+1)$ *or* $n \not\equiv 0 \bmod (r+1)$.

Proof. Since $\mathrm{Asym}(C) = \vec{C}_n$ and in virtue of Theorem 2.3 we have only to prove that C has a kernel iff $n \equiv 0 \bmod (r+1)$. If $n \equiv 0 \bmod (r+1)$, $\{i \mid i \equiv 0 \bmod (r+1)\}$ is a kernel of C. Let N be a kernel of C. If $u \in N$, $u' = u + 1 \notin N$. Take $u' + j \in N$ such that $u(u'+j) \in F(C)$. Clearly $u' + k \notin N$ for $k = \pm 2, \ldots, \pm(r-1), -r$, since for these values of k, $u' + k$ is adjacent to u. Then $j = r$ and $u + r + 1 \in N$. Therefore $u + m(r+1) \in N$ (operations taken mod n) for every m. Since N is an independent set, we must have $n \equiv 0 \bmod (r+1)$. In particular we have

Corollary 2.3. $\vec{C}_n(1, \pm 2, \ldots, \pm[\frac{1}{2}n])$ *is an* R^-*-digraph for* $n \geqslant 4$.

Another direct consequence of Theorem 2.2 is

Theorem 2.5. *D is an R-digraph iff for every strong component* α *of* $\mathrm{Asym}(D)$, $D[V(\alpha)]$ *is an R-digraph.*

Proof. Let H be an induced R^--subdigraph of D. By Theorem 2.2, $\mathrm{Asym}(H)$ is strongly connected and thus it is contained in a strong component α of $\mathrm{Asym}(D)$. Therefore H is an induced subdigraph of $D[V(\alpha)]$. This gives a contradiction. The converse is obvious. □

It is well known that any bipartite digraph is R-digraph. As an application of Theorem 2.5 we obtain

Corollary 2.4. *If for every strong component* α *of* $\mathrm{Asym}(D)$, $D[V(\alpha)]$ *is bipartite, then D is an R-digraph.*

Corollary 2.4 includes as a particular case the following result due to Duchet.

Theorem 2.6 (Duchet [3]). *If every directed cycle of odd length in D has at least two symmetric arcs, D is an R-digraph.*

Proof. It is sufficient to prove the following

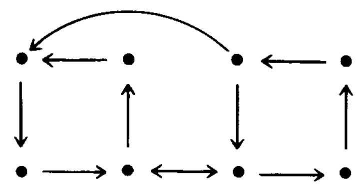

Fig. 1

Lemma 2.1. *If every directed cycle of odd length in D has at least two symmetric arcs, then for every strong component α of* Asym(D), $D[V(\alpha)]$ *is bipartite.*

Proof. Let α be a strong component of Asym(D).

Since α is strongly connected and does not contain directed cycles of odd length, α is bipartite [9, Theorem 6.14]. Therefore $D[V(\alpha)]$ is bipartite for otherwise a directed cycle of odd length with only one symmetric arc would be obtained. □□

The example shown in Fig. 1 (which is an R-digraph, by Corollary 2.4) shows that Corollary 2.4 is strictly stronger than Theorem 2.6.

3. R^--digraphs whose asymmetrical part is separable

The main result of this section is Theorem 3.4. The following lemma can be easily proved.

Lemma 3.1. *Let D_1, D_2 and D be digraphs, $v \in V(D_i)$, $i = 1, 2$. Suppose that $D_1 \cup D_2 = D$, $V(D_1) \cap V(D_2) = \{v\}$ and N_1 is a kernel of D_1.*

(i) *If N_2 is a kernel of D_2 and $v \in (N_1 \cap N_2) \cup (N_1^c \cap N_2^c)$, then $N_1 \cup N_2$ is a kernel of D.*

(ii) *If $v \notin N_1$ and N_2' is a kernel of $D_2 - v$, then $N_1 \cup N_2'$ is a kernel of D.*

Lemma 3.2. *Let D_1, D_2 and D be digraphs, $v \in V(D_i)$, $i = 1, 2$. Suppose that $D_1 \cup D_2 = D$, $V(D_1) \cap V(D_2) = \{v\}$, N is a kernel of D and $N_i = N \cap V(D_i)$, $i = 1, 2$. If N_2 is not a kernel of D_2, then N_1 is a kernel of D_1 and N_2 is a kernel of $D_2 - v$.*

Proof. If $v \in N$, then N_i is a kernel of D_i for $i = 1, 2$. If $v \notin N$, there exists a vN_1-arc and no vN-arc since otherwise N_2 would be a kernel of D_2 and Lemma 3.2 follows. □

Lemma 3.3. *Let D_1 and D_2 be digraphs, $v \in V(D_i)$, $i = 1, 2$. Suppose that $V(D_1) \cap V(D_2) = \{v\}$, $u_iv \in$ Sym(D_i), $H_i = D_i - u_iv - vu_i$, $i = 1, 2$ and $D = (H_1 \cup$

$H_2) + u_1u_2 + u_2u_1$. *If each one of* H_i, $H_i - v$, $H_i - \{u_i, v\}$, $(i = 1, 2)$, $H_1 - u_1$ *and* D_2 *has a kernel, then* D *has a kernel.*

Proof. In what follows, $N(S)$ denotes in general any kernel of S. Suppose that D has no kernel. Then

(i) $\{u_1, u_2\} \subset N(H_1 \cup H_2)$ for every kernel $N(H_1 \cup H_2)$ of $H_1 \cup H_2$. For otherwise $N(H_1 \cup H_2)$ would be a kernel of D. Let $N(H_1)$ be a kernel of H_1 and suppose that $v \notin N(H_1)$. Thus $u_1 \in N(H_1)$ for otherwise $N = N(H_1) \cup N(H_2 - v)$ would be a kernel of $H_1 \cup H_2$ not containing u_1 in contradiction with (i). Then $N' = N(H_1) \cup N(H_2 - \{u_2, v\})$ is a kernel of $H_1 \cup H_2$ and, since $u_2 \notin N'$ also a kernel of D which is in contradiction with the initial assumption. Therefore $v \in N(H_1)$ and similarly $v \in N(H_2)$. It follows that $N(H_1) \cup N(H_2)$ is a kernel of $H_1 \cup H_2$ and by (i) $u_i \in N(H_i)$. We have proved

(ii) u_i, $v \in N(H_i)$ for every kernel $N(H_i)$ of H_i, $i = 1, 2$.

Let $N(D_2)$ be a kernel of D_2. Then $v \notin N(D_2)$ for otherwise $N = N(D_2) \cup N(H_1)$ would be a kernel of D. So $v \notin N(D_2)$ for every kernel $N(D_2)$ of D_2. Furthermore $u_2 \in N(D_2)$ since otherwise $N(D_2)$ would be a kernel of H_2 not containing v. We have proved

(iii) Every Kernel $N(D_2)$ of D_2 satisfies $v \notin N(D_2)$ and $u_2 \in N(D_2)$.

(iv) Vertex v belongs to every kernel $N(H_1 - u_1)$ of $H_1 - u_1$.

Since otherwise by (iii), $N(H_1 - u_1) \cup N(D_2)$ would be a kernel of D. Finally by using (ii) and (iv) we conclude that $N(H_1 - u_1) \cup N(H_2)$ is a kernel of D which gives the final contradiction. □

Lemma 3.4. *Let* D_1, D_2 *be digraphs,* $v \in V(D_i)$, $i = 1, 2$. *Suppose that* $V(D_1) \cap V(D_2) = \{v\}$, $u_iv \in \mathrm{Sym}(D_i)$, $H_i = D_i - u_iv - vu_i$, $i = 1, 2$ *and* $D = (H_1 \cup H_2) + u_1u_2 + u_2u_1$. *If* N *is a kernel of* D *and* $N_i = N \cap V(D_i)$, $i = 1, 2$, *then either* N_1 *is a kernel of* D_1 *or* N_2 *is a kernel of* D_2.

Proof. Let N be a kernel of D, $N_i = N \cap V(D_i)$, $i = 1, 2$. Obviously $\{u_1, u_2\} \not\subset N$. W.l.o.g. we can assume that $u_1 \notin N$. If $v \in N$, N_1 is a kernel of D_1; if $v \notin N$ and $u_2 \in N$, N_2 is a kernel of D_2. Finally, in case $v \notin N$ and $u_2 \notin N$, N_i is a kernel of D_i provided that there exists a vN_i-arc. This is clearly true for some index i. □

In what follows we need the following result due to Jacob.

Theorem 3.1 (Jacob [10, pp. 78–82]). *Let* D_1, D_2 *and* D *be digraphs such that* $V(D_1) \cap V(D_2) = \{v\}$ *and* $D = D_1 \cup D_2$. *Then* D *is an* R*-digraph iff* D_1 *and* D_2 *are* R*-digraphs.*

Theorem 3.1 is a direct consequence of Lemma 3.1.

Theorem 3.2. *Let D_1, D_2, D, H_1 and H_2 be as in Lemma* 3.4. *Suppose that H_1 and H_2 are R-digraphs. Then D is an R^--digraph iff D_1 and D_2 are R^--digraphs.*

Proof. (i) Suppose that D_1 and D_2 are R^--digraphs. By Lemma 3.4, D has no kernel. Let $D' \not\subseteq^* D$. If $\{v, u_1, u_2\} \not\subset V(D')$, D' has a kernel by Theorem 3.1. If $\{v, u_1, u_2\} \subset V(D')$, D' has a kernel by Lemma 3.3. It follows that D is an R^--digraph.

(ii) Suppose that D is an R^--digraph. By Lemma 3.3, D_i has no kernel for $i = 1, 2$. Let $D_2' \not\subseteq^* D_2$. If $\{v, u_2\} \not\subset D_2'$, $D_2' \not\subseteq^* D$ and therefore D_2' has a kernel. If $\{v, u_2\} \subset D_2'$, let $D' = D[V(D_2') \cup V(D_1)]$. Since $D' \not\subseteq^* D$, D' has a kernel and by using Lemma 3.4 and the fact that D_1 has no kernel we conclude that D_2' has a kernel. □

Theorem 3.3 can be obtained applying Theorem 3.1 and Lemmas 3.3 and 3.4.

Theorem 3.3. *Let D_1, D_2, H_1, H_2 and D be as in Lemma* 3.4. *If H_1 and H_2 are R-digraphs, then D is an R-digraph iff at least one of D_1 and D_2 is an R-digraph.*

Definition 3.1. Let D be a digraph such that $vu, uv \in F(D)$, D is said to be an $R^-(u, v)$-digraph iff D is an R^--digraph and $D - uv - vu$ is an R-digraph.

From Theorems 3.1 and 3.2 we can easily prove Theorem 3.4.

Theorem 3.4. *Let D_1, D_2 and D be as in Lemma* 3.3. *If D_i is an $R^-(u_i, v)$-digraph for $i = 1, 2$, then D is an $R^-(u_1, u_2)$-digraph.*

Example 1. Let $D = \vec{C}_n(1, \pm 2, \ldots, \pm r)$, $n \not\equiv 0 \,(\mathrm{mod}(r+1))$. By Theorem 2.4, D is an R^--digraph. Let i, j, k be integers modulo $r+1$ such that $k = j + i$, $i = 2, \ldots, r$. Then D is not an $R^-(j, k)$-digraph only in case $i = r$, $n \not\equiv r \,(\mathrm{mod}(r+1))$ and $n \geqslant 2r + 1$.

Proof. By symmetry we can assume $k = 0$; therefore $j = n - i$. Any kernel N of $D_1 = D - j0 - 0j$ must contain 0 and j, for otherwise N would be a kernel of D. If $n \equiv i \,\mathrm{mod}(r+1)$, $\{k(r+1) \mid 0 \leqslant k \leqslant (n-i)/(r+1)\}$ is a kernel of D_1 and by Theorem 2.3, D_1 is an R-digraph. If $n \not\equiv i \,\mathrm{mod}(r+1)$ and $i \not\equiv r$, take $A = \{r+1, r+2, \ldots, n-i-r-1\}$ whenever $n \geqslant 2(r+1) + i$ and otherwise $A = \emptyset$ and let $N(D[A])$ be a kernel of $D[A]$. It is easy to see that $N(D[A]) \cup \{0, n-i\}$ is a kernel of D_1. In case $n \not\equiv i \,\mathrm{mod}(r+1)$ and $i = r$:

(a) if $n \geqslant 2r + 1$ proceed as in the proof of Theorem 2.4 to conclude that D_1 is an R^--digraph;

(b) if $n = 2r$, $\{0, r\}$ is a kernel of D_1.

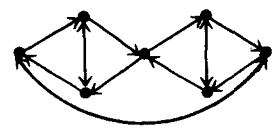

Fig. 2

Example 2. Taking $D_1 \cong D_2 \cong C_4(1, \pm 2)$, apply Lemma 3.3 to get the digraph D of Fig. 2 ($u_i v$, vu_i are the arcs corresponding to a diagonal of $\vec{C}_4$). Using Theorem 3.4 and the preceding paragraph we conclude that D is an R^--digraph. In fact, D is an example of non Hamiltonian R^--digraph. Starting with D and using repeatedly the operation $\alpha(G, v)$ introduced in [4]. We can obtain examples of non Hamiltonian R^--oriented graphs.

4. Dichromatic number and quasi R-digraphs

4.1. *Some constructions*

The constructions given in this section are useful to enlarge the class of known R-digraphs and R^--digraphs.

4.1.1. If $f = uv \in F(D)$, $D(f/P_n)$ will denote any digraph D' such that $D' = (D - f) \cup P_n(u, v)$, where $P_n(u, v)$ is a uv-directed path of length $n - 1$ satisfying $V(P_n(u, v) \cap D) = \{u, v\}$.

Theorems 4.1 and 4.2 were found independently by Galeana-Sánchez and Neumann-Lara [6] and Duchet and Meyniel [4].

Theorem 4.1. *$D(f/P_{2k})$ has a kernel iff D has a kernel.*

Proof. Any kernel of D can be extended to a kernel of $D(f/P_{2k})$. If N' is a kernel of $D(f/P_{2k})$, $N' \cap V(D)$ is a kernel of D. □

By using Theorems 3.1 and 4.1 we can prove

Theorem 4.2. *Suppose that $D - f$ is an R-digraph. Then $D(f/P_{2k})$ is an R-digraph (resp. R^--digraph) iff D is an R-digraph (resp. R^--digraph).*

Remark 4.1. By Theorem 4.1 it is sufficient to prove that $D(f/P_{2k})$ is a quasi R-digraph iff D is a quasi R-digraph.

We omit the proof of Theorem 4.3 which is a simple generalization of Theorem 4.2.

Theorem 4.3. *Let $\emptyset \neq F_0 \subset F(D)$ and D' a digraph obtained from D by replacing each $uv \in F_0$ by a uv-directed path $P(u, v)$ of odd length so that $V(D \cap P(u, v)) = \{u, v\}$ and $V(P(u, v) \cap P(u', v')) = \{u, v\} \cap \{u', v'\}$ whenever $uv \neq u'v'$, $uv, u'v' \in F_0$. Suppose furthermore that $D - F'$ is an R-digraph provided $\emptyset \neq F' \subset F_0$. Then D' is an R-digraph (resp. R^--digraph) iff D is an R-digraph (resp. R^--digraph).*

4.1.2. Let D be a digraph and $\alpha = (\alpha_u)_{u \in V(D)}$ a family of non empty mutually disjoint digraphs. The diagraph $\sigma(D, \alpha)$ is defined by:

(i) $V(\sigma(D, \alpha)) = \bigcup_{u \in V(D)} V(\alpha_u)$;

(ii) $w_1 w_2 \in F(\sigma(D, \alpha))$ iff

(1) $w_1, w_2 \in V(\alpha_u)$ and $w_1 w_2 \in F(\alpha_u)$ for some $u \in V(D)$, or

(2) $w_1 \in V(\alpha_u)$, $w_2 \in V(\alpha_v)$ and $uv \in F(D)$ for some $u, v \in V(D)$.

Remark 4.2. Notice that if α_u is isomorphic to α^0 for every $u \in V(D)$, $\sigma(D, \alpha)$ is isomorphic to the lexicographic product $D[\alpha^0]$. Moreover if D and the α_u's are oriented graphs, $\sigma(D, \alpha)$ is an oriented graph.

Theorem 4.4. *$\sigma(D, \alpha)$ is an R-digraph iff D and every α_u are R-digraphs.*

Proof. Since α_u and D are induced subdigraphs of $\sigma(D, \alpha)$ it follows that D and α_u are R-digraphs provided $\sigma(D, \sigma)$ is an R-digraph. The converse follows by observing that every induced subdigraph of $\sigma(D, \alpha)$ has the form $\sigma(D', \alpha')$, where $D' \subset^* D$ and $\alpha'_u \subset^* \alpha_u$ for $u \in V(D')$, and therefore $\bigcup_{u \in K} Q_u$ is a kernel of D' whenever K is a kernel of D' and Q_u is a kernel of α_u for each $u \in V(D')$. □

Corollary 4.1. *$D[\alpha^0]$ is an R-digraph iff D and α^0 are R-digraphs.*

4.2. *Dichromatic number and kernel theory*

The dichromatic number $d_k(D)$ of D was defined in [13] (see also [5]), and independently in [11] as the minimum number of colours required to colour the vertices of D in such a way that the chromatic classes induce acyclic subdigraphs of D. Clearly $d_k(D) \geq \chi(D)$. The dichromatic number is a generalization of the chromatic number. In particular, they coincide for symmetric digraphs. It was proved in [13] that $d_k(D) \leq 2$ for any digraph D not containing directed cycles of odd length. Therefore Richardson's theorem is useful only for digraphs whose dichromatic number is less or equal to 2. According to Corollary 2.3 there exist R^--digraphs (and obviously R-digraphs) with arbitrarily large dichromatic number; more precisely $d_k(\vec{C}_n(1, \pm 2, \ldots, \pm[\frac{1}{2}n])) = [\frac{1}{2}n]^*$.

For oriented graphs the situation is more complicated: The only tournament which is an R^--digraph is the triangle (directed cycle of length 3); therefore a tournament T is an R-digraph iff T is acyclic or equivalently, $d_k(T) = 1$. It was proved in [13] that $d_k(D[\alpha]) \geqslant d_k(D) + d_k(\alpha) - 1$. Then $d_k(D[\vec{C}_4]) \geqslant d_k[D] + 1$ since $d_k(\vec{C}_4) = 2$. From this inequality and Remark 4.2, it turns out that R-oriented graphs with arbitrarily large dichromatic number can be constructed. In a forthcoming paper [8] we shall prove the following result.

Theorem 4.5. *For every R-digraph (resp. R-oriented graph) D_0 there exists an R^--digraph (resp. R^--oriented graph) D such that $D_0 \subset^* D$.*

This theorem implies that there exist R^--oriented graphs with arbitrarily large dichromatic number and consequently with arbitrarily large chromatic number.

References

[1] C. Berge, Graphs and Hypergraphs (North-Holland, Amsterdam, 1973).
[2] P. Duchet, Representation; noyaux en theorie des graphes et hypergraphes, Thèse, Paris (1979).
[3] P. Duchet, Graphes Noyau-parfaits, Ann. Discrete Math. 9 (1980) 93–101.
[4] P. Duchet and H. Meyniel, A note on kernel-critical graphs, Discrete Math. 33 (1981) 103–105.
[5] P. Erdös, Problems and results in Number Theory and Graph Theory, Proc. Ninth Manitoba Conference on Numerical Math. and Computing (1979) 3–21.
[6] H. Galeana-Sánchez, A counterexample to a conjecture of Meyniel on kernel-perfect graphs, Discrete Math. 41 (1982) 105–107.
[7] H. Galeana-Sánchez and V. Neumann-Lara, On kernels and semikernels of digraphs, Discrete Math. 48 (1984) 67–76.
[8] H. Galeana-Sánchez and V. Neumann-Lara, Extending kernel-perfect digraphs to kernel-perfect critical digraphs, preprint.
[9] F. Harary, R.Z. Norman and D. Cartwright, Structural Models (Wiley, New York, 1965).
[10] H. Jacob, Etude theórique du Noyau d' un graphe, Thèse, Université Pierre et Marie Curie, Paris VI (1979).
[11] H. Meyniel, Extension du nombre chromatique et du nombre de stabilité, preprint.
[12] V. Neumann-Lara, Seminúcleos de una digráfica, An. Inst. Mat. Univ. Nac. Autónoma México II (1971).
[13] V. Neumann-Lara, The dichromatic number of a digraph, J. Combin. Theory Ser. B 33 (3) (1982) 265–270.
[14] M. Richardson, On weakly ordered systems, Bull. Amer. Math. Soc. 52 (1946) 113.
[15] M. Richardson, Solutions of irreflexive relations, Ann. Math. (2) 58 (1953) 573.
[16] M. Richardson, Extension theorems for solutions of irreflexive relations, Proc. Nat. Acad. Sci. U.S.A. 39 (1953) 649.
[17] J. Von Neumann and O. Morgenstern, Theory of Games and Economic Behavior (Princeton Univ. Press, Princeton).