16/08/16

Steffen Oppermann:
d-tilting bundles for Geigle-Lenzing weighted projective spaces $\mathbb{P}^d$

jt. M. Herschend, O. Iyama, H. Minamoto

Classically: Rep. theory for hereditary fd. algebras
short exact seq., AR quiver, ...

d-dimensional rep. theory [Iyama]
consider $(d+2)$-term exact seq $\in \operatorname{Ext}^d$, gldim $= d$
look subcat s.t. $\operatorname{Ext}^1, \ldots, \operatorname{Ext}^{d-1} = 0$

classically:

rep. finite | tame | wild

d-dimensional: ok ?

d-rep. finite | $\boxed{\text{today}}$

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{d-rep. infinite}}$

classically:

$\{$tame hereditary alg$\} \xleftrightarrow{\sim\ \text{der}} \{$Fano Geigle-Lenzing weighted proj lines$\}$

GL-weighted projective $\mathbb{P}^d$

today: hypersurface case $(d+2)$ weights

$$R = k[x_1, \dots, x_{d+2}] / (x_1^{p_1} + x_2^{p_2} + \dots + x_{d+2}^{p_{d+2}}), \quad p_i \geq 2$$

graded over $\mathbb{L} = \langle \vec{x}_1, \dots, \vec{x}_{d+2} \rangle / (p_1\vec{x}_1 = p_2\vec{x}_2 = \dots = \underbrace{p_{d+2}\vec{x}_{d+2}}_{= \vec{c}})$

consider categories $\mathrm{mod}^{\mathbb{L}} R$, $\mathrm{CM}^{\mathbb{L}} R$

$$\mathrm{coh}\, \mathbb{X} = \mathrm{mod}^{\mathbb{L}} R / \underbrace{\mathrm{mod}_0^{\mathbb{L}} R}_{\text{fl modules}}$$

GL-weighted proj $\mathbb{P}^d$

$$\vec{\omega} = \vec{c} - \sum_{i=1}^{d+2} \vec{x}_i \in \mathbb{L}$$

gives Serre duality

$$\mathrm{Ext}^i_{\mathbb{X}}(F, G) = D\,\mathrm{Ext}^{d-i}(G, F(\vec{\omega}))$$

$$\mathbb{X} \text{ Fano} \overset{\text{def}}{\iff} \exists\, n \gg 0 : n \cdot \vec{\omega} > 0$$

$$\iff 1 - \sum \frac{1}{p_i} < 0$$

Hope: $\mathbb{X}$ Fano $\Rightarrow \exists\, T$ tilting bundle s.t.

$\mathrm{End}_{\mathbb{X}}(T)$ is d-tame

enough [Buchweitz, Hille] that $\underbrace{\mathrm{gldim}\, \mathrm{End}_{\mathbb{X}}(T) \leq d}_{\text{def. for } \underline{d\text{-tilting}}}$

Proposition: $\dfrac{\mathrm{vect}\,\mathbb{X}}{\mathrm{line}\,\mathbb{X}} \longleftrightarrow \underline{\mathrm{CM}}^{\mathbb{L}} R$

Proposition: $\underline{\mathrm{CM}}^{\mathbb{L}} R$ has a tilting object

with $\mathrm{End} = \bigotimes_{i=1}^{d+2} k\vec{A}_{p_i-1}$

If weight sequence contains $(2,2)$ or $(2,3,p)$

or $(3,3,p,q)$ for $p,q \in \{3,4,5\}$ then

$\underline{\mathrm{CM}}^{\mathbb{L}} R$ has a d-tilting object.

Def: $\gamma : \mathbb{L} \to \mathbb{Q}$ is $\underline{(\vec{\omega}-)\text{normal}}$ if

$$\left.\begin{array}{l} \left.\begin{array}{l} 1)\ \gamma(\vec{x}) \leq \gamma(\vec{y}) \quad \text{if } \vec{x} \leq \vec{y} \\ 2)\ \gamma(\vec{x}+\vec{c}) = \gamma(\vec{x}) + 1 \end{array}\right\} \text{normal} \\ \left[\ 3)\ \gamma(\vec{x}+\vec{\omega}) \leq \gamma(\vec{x})\ \right] \end{array}\right\} \vec{\omega}\text{-normal}$$

Def: $M \in \underline{\mathrm{CM}}^{\mathbb{L}} R$ has a $\underline{\gamma\text{-resolution}}$ if

$$\bigoplus_{\gamma(\vec{x})=0} R(\vec{x})^{?} \longrightarrow M \hookrightarrow \bigoplus_{\gamma(\vec{y})=\frac{1}{2}} R(\vec{y})^{?}$$

Theorem: $M \in \underline{CM}^L R$ d-tilting and
$\gamma : \mathbb{L} \to \mathbb{Q}$ $\vec{\omega}$-normal s.t.
M has a γ-resolution then
M lifts to a d-tilting bundle.

Strategy for finding γ:

- consider (very) small dimension [$d = -1, 0$]
- use tensor products [Yoshino]

Theorem: If the weight sequence contains
$(2,2)$ or $(2,3,p)$ or $(3,3,p,q)$ for $p,q \in \{3,4,5\}$
then $\mathbb{X}$ has a d-tilting bundle.