

16/08/16

Steffen Oppermann:

# d-tilting bundles for Geigle-Lenzing weighted projective spaces <sup>pd</sup>

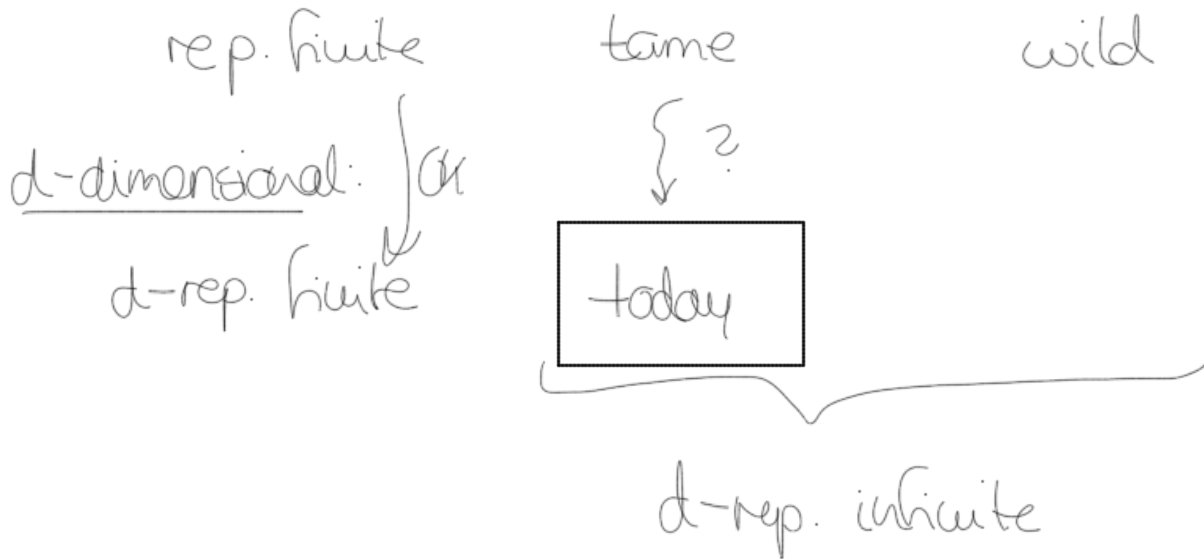
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classically: Rep. theory for hereditary fd. algebras  
short exact seq., AR quiver, ...

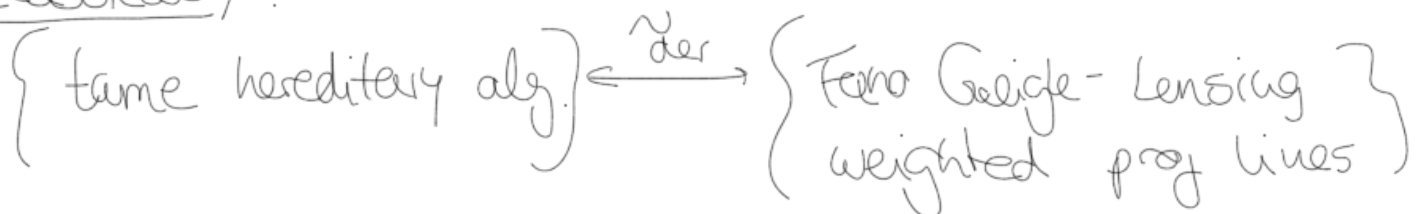
d-dimensional rep. theory [Iyama]

consider (d+1)-term exact seq  $\in \text{Ext}^d$ ,  $\text{gldim} = d$   
look subcat s.t.  $\text{Ext}^1, \dots, \text{Ext}^{d-1} = 0$

classically:



classically:



# GL-weighted projective $\mathbb{P}^d$

today: hypersurface case  $(d+2)$  weights

$$R = k[x_1, \dots, x_{d+2}] / (x_1^{p_1} + x_2^{p_2} + \dots + x_{d+2}^{p_{d+2}}), \quad p_i \geq 2$$

graded over  $\mathbb{L} = \langle \vec{x}_1, \dots, \vec{x}_{d+2} \rangle / (p_1 \vec{x}_1 = p_2 \vec{x}_2 = \dots = p_{d+2} \vec{x}_{d+2})$

consider categories  $\text{mod}^{\mathbb{L}} R, \text{CM}^{\mathbb{L}} R$

$$\text{Coh } X = \text{mod}^{\mathbb{L}} R / \underbrace{\text{mod}_0^{\mathbb{L}} R}_{\text{fl modules}}$$

GL-weighted proj  $\mathbb{P}^d$

$$\vec{\omega} = \vec{z} - \sum_{i=1}^{d+2} \vec{x}_i \in \mathbb{L}$$

gives Serre duality

$$\text{Ext}_X^i(F, G) = D \text{Ext}^{d-i}(G, F(\vec{\omega}))$$

$$\begin{aligned} X \text{ Fano} &\stackrel{\text{def}}{\iff} \exists n \gg 0 : n \cdot \vec{\omega} > 0 \\ &\iff 1 - \sum \frac{1}{p_i} < 0 \end{aligned}$$

Hope:  $X \text{ Fano} \Rightarrow \exists T$  tilting bundle s.t.

$\text{End}_X(T)$  is  $d$ -tame

enough [Bocklandt, Hille] that  $\text{gldim } \text{End}_X(T) = d$

def. for  $d$ -tilting

Proposition:  $\frac{\text{Vect } X}{\text{line } X} \longleftrightarrow \underline{\text{CM}}^d R$

Proposition:  $\underline{\text{CM}}^d R$  has a tilting object

with  $\text{End} = \bigotimes_{i=1}^{d+2} k\vec{A}_{p_i-1}$

If weight sequence contains  $(2,2)$  or  $(2,3,p)$  or  $(3,3,p,q)$  for  $p,q \in \{3,4,5\}$  then  $\underline{\text{CM}}^d R$  has a  $d$ -tilting object.

Def:  $\gamma: \mathbb{L} \rightarrow \mathbb{Q}$  is  $(\vec{w}-)$ normal if

1)  $\gamma(\vec{x}) \leq \gamma(\vec{y})$  if  $\vec{x} \leq \vec{y}$

2)  $\gamma(\vec{x} + \vec{c}) = \gamma(\vec{x}) + 1$

[ 3)  $\gamma(\vec{x} + \vec{w}) \leq \gamma(\vec{x})$  ]

} normal

$\vec{w}$ -normal

Def:  $M \in \underline{\text{CM}}^d R$  has a  $\gamma$ -resolution if

$$\bigoplus_{\gamma(\vec{x})=0} R(\vec{x})^? \rightarrow M \leftarrow \bigoplus_{\gamma(\vec{y})=\frac{1}{2}} R(\vec{y})^?$$

Theorem:  $M \in \underline{CM}^d R$   $d$ -tilting and  
 $f: L \rightarrow Q$   $\bar{w}$ -normal s.t  
 $M$  has a  $f$ -resolution then  
 $M$  lifts to a  $d$ -tilting bundle.

Strategy for finding  $f$ :

- consider (very) small dimension  $[d = -1, 0]$
- use tensor products [Yoshino]

Theorem: If the weight sequence contains

$(2, 2)$  or  $(2, 3, p)$  or  $(3, 3, p, q)$  (or  $p, q \in \{3, 4, 5\}$ )  
 then  $X$  has a  $d$ -tilting bundle.