Newton's method for solving differential equations

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Exercise 1 *Using the Newton polygon, compute at least five terms of a series solution $y(x)$ for the following equations.*

1. $f(x, y) = x + y^2 + xy = 0$
2. $f(x, y) = y^3 + xy + x^2 = 0$

Exercise 2 *Using the Newton polygon, compute at least three terms of a series solution $y(x)$ for the following polynomials.*

1. $f(x, y) = y^4 - 2y^2x^3 + x^6 - 4x^5y - x^7 = 0$
2. $f(x, y) = y^4 - x^9 - 4x^7y = 0$

Exercise 3 *Consider the Pfaffian equations given by*

$$\omega_1 = 2ydx - 3xdy,$$

and

$$\omega_2 = \left(y^3 - x^2y\right) dx + \left(x^3 - 2xy^2\right) dy, \tag{1}$$

Show that there are infinite solutions of the form $y(x) = cx^{\frac{3}{2}} + \cdots$, with $c \in \mathbb{C} \setminus \{0\}$.

Exercise 4 *Consider the Pfaffian equation given by*

$$\omega = \left(y^4 - 8x^7y\right) dx + \left(4x^8 - 4y^3x\right) dy. \tag{2}$$

Show that there are solutions of the form $y(x) = c_\mu x^\mu + \cdots$, for certain $\mu \in \mathbb{Q}_{\geq 0}$. Consider $c_\mu = 1$ and compare with 2) of the exercise 2.

Exercise 5 *Let K be a field of characteristic p. Let $f \in K[x, y]$ given by*

$$f(x, y) = x + y + y^p$$

What happen if we apply the Newton method for compute $y(x)$?

Exercise 6 *Determine at least two terms of a series solution of the equation*

$$p(x, y, z) = 4x^2y + \left(x^2y + xy^2 + xy + y\right)^2 - z^2 = 0,$$

when solved for z.