1-academic-examples

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[1]:
```
from sympy import *
from DifferentialAlgebra import *
init_printing ()
```

0.1 First Example

[2]:
```
xi = var('xi')
x, y = function ('x, y')
```

Given y = a series in x, find an ODE for y. Independent variables cannot be eliminated:

```
define x as a differential indeterminate
rename the independent variable as xi
```

[3]:
```
sought_series = xi**2
sought_series
```

[3]: ξ^2

[4]:
```
syst = [Eq (y(xi),x(xi)**2), Eq(Derivative(x(xi),xi),1)]
syst
```

[4]: $\left[y(\xi) = x^2(\xi),\ \frac{d}{d\xi}x(\xi) = 1\right]$

The blocks list provides the ranking: x > y (eliminate x)

[6]:
```
R = DifferentialRing (derivations = [xi], blocks = [x,y])
R
```

[6]: differential_ring

[8]:
```
ideal = R.RosenfeldGroebner (syst)
ideal
```

[8]: [regular_differential_chain]

[9]:
```
A = ideal[0]
A.equations (solved=True)
```

[9]:
$$\left[\left(\frac{d}{d\xi}y(\xi)\right)^2 = 4y(\xi),\ x(\xi) = \frac{\frac{d}{d\xi}y(\xi)}{2}\right]$$

p is the sought ODE

[10]:
```
p = A.equations ()[0]
p
```

[10]:
$$-4y(\xi) + \left(\frac{d}{d\xi}y(\xi)\right)^2$$

[11]:
```
R.evaluate (p, {y(xi):sought_series})
```

[11]:
$$-4\xi^2 + \left(\frac{d}{d\xi}\xi^2\right)^2$$

[12]:
```
R.evaluate (p, {y(xi):xi**2}).doit ()
```

[12]:
$$0$$

0.2 Second Example

[13]:
```
xi = var('xi')
x, y, z = indexedbase ('x, y, z')
```

[14]:
```
sought_series = xi**2 + xi**(3/Integer(2))
sought_series
```

[14]:
$$\xi^{\frac{3}{2}} + \xi^2$$

The notation used for differential equations is the ‘jet’ one

[15]:
```
syst = [Eq(y,x**2 + z), Eq(z**2,x**3), x[xi] - 1]
syst
```

[15]:
$$\left[y = x^2 + z,\ z^2 = x^3,\ x_{\xi} - 1\right]$$

The blocks list provides the ranking: (z,x) > y

[16]:
```
R = DifferentialRing (derivations = [xi], blocks = [[z,x],y], notation = 'jet')
R
```

[16]: differential_ring

[17]:
```
ideal = R.RosenfeldGroebner (syst)
ideal
```

[17]: [regular_differential_chain]

[18]:
```
A = ideal[0]
A.equations (solved=True)
```

[18]:

$$\left[y_\xi^4 = \frac{y_\xi^3}{2} + 8y_\xi^2 y - 16y^2 - \frac{27y}{16},\ x = \frac{-192y_\xi^3 - 12y_\xi^2 + 1280y_\xi y + 720y}{1024y - 27},\ z = \frac{216y_\xi^3 + 512y_\xi^2 y - 1440y_\xi y - 2048y^2 -}{1024y - 27}\right.$$

[20]:
```
p = A.equations ()[0]
p
```

[20]:

$$16y_\xi^4 - 8y_\xi^3 - 128y_\xi^2 y + 256y^2 + 27y$$

[21]:
```
simplify (R.evaluate(p, {y:sought_series}).doit())
```

[21]:

$$0$$

[ ]:
```

```