5-input-output-relation

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[1]:
```
from sympy import *
from DifferentialAlgebra import *
init_printing ()
```

[2]:
```
t, k12, k21, Ve, ke = var ('t, k_12, k_21, V_e, k_e')
params = [k12, k21, ke, Ve]
params
```

[2]:

$$[k_{12},\ k_{21},\ k_e,\ V_e]$$

[3]:
```
x1, x2, y, u = function ('x_1, x_2, y, u')
```

[4]:
```
R = DifferentialRing (derivations = [t], blocks = [y, x1, x2, u, params],
                      parameters = params)
```

[5]:
```
syst = [ Eq(Derivative(x1(t),t), -k12*x1(t) + k21*x2(t) - Ve*x1(t)/(ke+x1(t)) +
 ↪u(t)),
         Eq(Derivative(x2(t),t), k12*x1(t) - k21*x2(t)),
         Eq(y(t),x1(t))]
A_R = RegularDifferentialChain (syst, R)
A_R.equations (solved=True)
```

[5]:

$$\left[\mathrm{x}_1(t) = -\frac{-k_{21}\,\mathrm{x}_2(t) - \frac{d}{dt}\mathrm{x}_2(t)}{k_{12}},\ \frac{d}{dt}\mathrm{x}_1(t) = \frac{-V_e\,\mathrm{x}_1(t) - k_{12}k_e\,\mathrm{x}_1(t) - k_{12}\,\mathrm{x}_1{}^2(t) + k_{21}k_e\,\mathrm{x}_2(t) + k_{21}\,\mathrm{x}_1(t)\,\mathrm{x}_2(t)\right.}{k_e + \mathrm{x}_1(t)}$$

[6]:
```
S = DifferentialRing (derivations = [t],
                      blocks = [x1, x2, y, u, params],
                      parameters = params)
```

[7]:
```
A_S = A_R.change_ranking (S)
A_S.equations (solved=True)
```

[7]:

$$\left[\frac{d^2}{dt^2}y(t) = \frac{-V_ek_{21}k_ey(t) - V_ek_{21}y^2(t) - V_ek_e\frac{d}{dt}y(t) - k_{12}k_e^2\frac{d}{dt}y(t) - 2k_{12}k_ey(t)\frac{d}{dt}y(t) - k_{12}y^2(t)\frac{d}{dt}y(t) + k_{21}k_e^2u(t)\right.}{}$$

[8]:
```
leader = var ('leader')
IO_rel = A_S.equations (selection = Eq(leader,Derivative(y(t),t,t)))[0]
```

```
IO_rel
```

[8]:

$$V_e k_{21} k_e y(t) + V_e k_{21} y^2(t) + V_e k_e \frac{d}{dt} y(t) + k_{12} k_e^2 \frac{d}{dt} y(t) + 2 k_{12} k_e y(t) \frac{d}{dt} y(t) + k_{12} y^2(t) \frac{d}{dt} y(t) - k_{21} k_e^2 u(t) + k_{21} k_e^2 \frac{d}{dt} y(t) - 2 k_{21} k_e u(t) y(t) + 2 k_{21} k_e y(t) \frac{d}{dt} y(t) - k_{21} u(t) y^2(t) + k_{21} y^2(t) \frac{d}{dt} y(t) - k_e^2 \frac{d}{dt} u(t) + k_e^2 \frac{d^2}{dt^2} y(t) - 2 k_e y(t) \frac{d}{dt} u(t) + 2 k_e y(t) \frac{d^2}{dt^2} y(t) - y^2(t) \frac{d}{dt} u(t) + y^2(t) \frac{d^2}{dt^2} y(t)$$

[9]:

```
IO_rel = IO_rel / S.initial (IO_rel)
IO_rel
```

[9]:

$$\frac{V_e k_{21} k_e y(t) + V_e k_{21} y^2(t) + V_e k_e \frac{d}{dt} y(t) + k_{12} k_e^2 \frac{d}{dt} y(t) + 2 k_{12} k_e y(t) \frac{d}{dt} y(t) + k_{12} y^2(t) \frac{d}{dt} y(t) - k_{21} k_e^2 u(t) + k_{21} k_e^2 \frac{d}{dt} y(t}{}$$

[10]:

```
IO_rel_integrated = S.integrate (IO_rel, t)
IO_rel_integrated
```

[10]:

$$\left[\frac{V_e k_{21} y(t) - k_{21} k_e u(t) - k_{21} u(t) y(t) - k_e \frac{d}{dt} u(t) - y(t) \frac{d}{dt} u(t)}{k_e + y(t)},\ \frac{-V_e k_e - k_{12} k_e^2 + k_{12} y^2(t) - k_{21} k_e^2 + k_{21} y^2(t)}{k_e + y(t)},\ y(t) \right]$$

[11]:

```
IO_rel_integrated = IO_rel_integrated[0] + \
                    Derivative (IO_rel_integrated[1], t) + \
                    Derivative (IO_rel_integrated[2], t, t)
IO_rel_integrated
```

[11]:

$$\frac{\partial}{\partial t} \frac{-V_e k_e - k_{12} k_e^2 + k_{12} y^2(t) - k_{21} k_e^2 + k_{21} y^2(t)}{k_e + y(t)} + \frac{d^2}{dt^2} y(t) + \frac{V_e k_{21} y(t) - k_{21} k_e u(t) - k_{21} u(t) y(t) - k_e \frac{d}{dt} u(t) - y(t) \frac{d}{dt}}{k_e + y(t)}$$

[12]:

```
simplify (IO_rel / S.initial (IO_rel) - IO_rel_integrated.doit())
```

[12]:

0

[ ]:

```

```