Ejercicio de Sistemas de Ecuaciones

Multiplicidad ≥ 2

Solucionar

$$\mathbf{Y} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix} \mathbf{Y}.$$

Solución.

Polinomio caracterśtico

$$q_A(t) = \begin{vmatrix} 1-t & 1 & 1 \\ 2 & 1-t & -1 \\ -3 & 2 & 4-t \end{vmatrix} = -(t-2)^3$$

Valor característico

$$\lambda = 2 \qquad \text{multiplicidad } = 3$$

Vectores característicos

$$A\xi = 2\xi$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

$$\begin{pmatrix} x+y+z \\ 2x+y-z \\ -3x+2y+4z \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix}$$

$$\begin{cases} x-y-z=0 \\ 2x-y-z=0 \\ 3x-2y-2z=0 \end{cases}$$

$\Rightarrow x = 0 \Rightarrow y = -z \Rightarrow$

$$\xi = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

dimensión geométrica $= 1$.

$$\mathbf{Y}_1 = e^{2x} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\eta = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\mathbf{Y}_2 = xe^{2x}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \eta e^{2x} = e^{2x}\begin{pmatrix} a \\ b+x \\ c-x \end{pmatrix}$$

Entonces

$$\mathbf{Y}_2' = (e^{2x}+2xe^{2x})\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + 2e^{2x}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = e^{2x}\begin{pmatrix} 2a \\ 1+2b+2x \\ -1+2c-2x \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix} e^{2x}\begin{pmatrix} a \\ b+x \\ c-x \end{pmatrix} = e^{2x}\begin{pmatrix} a+b+c \\ 2a+b+x-c+x \\ -3a+2b+2x+4c-4x \end{pmatrix}$$

$\Rightarrow$

$$\begin{cases} a-b-c=0 \\ 2a-b-c=1 \\ -3a+2b+2c=-1 \end{cases}$$

$\Rightarrow a=1 \Rightarrow b+c=1$. Fijamos $b=1 \Rightarrow c=0$.

$$\mathbf{Y}_2 = e^{2x}\begin{pmatrix} 1 \\ 1+x \\ -x \end{pmatrix}$$

$$\rho = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$\mathbf{Y}_3 = \frac{x^2}{2!}e^{2x}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + xe^{2x}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \rho e^{2x} =$$

$$= e^{2x}\begin{pmatrix} x+\alpha \\ \frac{x^2}{2}+x+\beta \\ -\frac{x^2}{2}+\gamma \end{pmatrix}$$

$$\mathbf{Y}_3' = 2e^{2x}\begin{pmatrix} x+\alpha \\ \frac{x^2}{2}+x+\beta \\ -\frac{x^2}{2}+\gamma \end{pmatrix} + e^{2x}\begin{pmatrix} 1 \\ x+1 \\ -x \end{pmatrix} = e^{2x}\begin{pmatrix} 2x+2\alpha+1 \\ x^2+3x+2\beta+1 \\ -x^2-x+2\gamma \end{pmatrix}$$

$$= e^{2x}\begin{pmatrix} 2x+\alpha+\beta+\gamma \\ 2(x+\alpha)+\left(\frac{x^2}{2}+x+\beta\right)+\frac{x^2}{2}-\gamma \\ -3(x+\alpha)+2\left(\frac{x^2}{2}+x+\beta\right)+4\left(-\frac{x^2}{2}+\gamma\right) \end{pmatrix}$$

$$= e^{2x} \begin{pmatrix} 2x + \alpha + \beta + \gamma \\ x^2 + 3x + 2\alpha + \beta - \gamma \\ -x^2 - x - 3\alpha + 2\beta + 4\gamma \end{pmatrix}$$

$\Rightarrow$

$$\begin{cases} \alpha - \beta - \gamma = -1 \\ 2\alpha - \beta - \gamma = 1 \\ 3\alpha - 2\beta - 2\gamma = 0 \end{cases}$$

$\Rightarrow \alpha = 2 \Rightarrow \beta + \gamma = 3$. Fijamos $\beta = 1 \Rightarrow \gamma = 2$.

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -3 & 2 & 4 \end{pmatrix}$$

$$\mathbf{Y}_3 = \frac{x^2}{2!} e^{2x} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + x e^{2x} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} e^{2x} = \begin{pmatrix} x + 2 \\ \frac{x^2}{2!} + x + 1 \\ -\frac{x^2}{2!} + 2 \end{pmatrix} e^{2x}$$