Sea V, W, Z espacios vectoriales sobre un campo F, todos de dimensión finita.

Sean $\beta \overset{o}{\hookrightarrow} V \qquad \beta \overset{o}{=} \{x_1, \ldots, x_m\}$

Sean $\gamma \overset{o}{\hookrightarrow} W \qquad \gamma \overset{o}{=} \{y_1, \ldots, y_n\}$

Sean $\delta \overset{o}{\hookrightarrow} Z \qquad \delta \overset{o}{=} \{z_1, \ldots, z_r\}$

Para cada $j = 1, \ldots, m$, existen $a_{j,1}, \ldots, a_{j,n} \in F$ tales que

$$T(x_j) = a_{1,j}y_1 + a_{2,j}y_2 + \ldots + a_{n,j}y_n$$

de donde

$$[T(x_j)]_\gamma = \begin{pmatrix} a_{1,j} \\ a_{2,j} \\ \vdots \\ a_{n,j} \end{pmatrix}.$$

Definimos

$$\begin{aligned} [T]_\beta^\gamma &= ([T(x_1)]_\gamma \; [T(x_2)]_\gamma \; \ldots \; [T(x_m)]_\gamma) \\ &= \begin{pmatrix} a_{1,1} & a_{1,2} & \ldots & a_{1,m} \\ a_{2,1} & a_{2,2} & \ldots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \ldots & a_{n,m} \end{pmatrix}. \end{aligned}$$

De igual forma tenemos que para cada $k = 1, \ldots, n$, existen $b_{k,1}, \ldots, b_{k,r} \in F$ tales que

$$U(y_k) = b_{1,k}z_1 + b_{2,k}z_2 + \ldots + b_{r,k}z_r$$

de donde

$$[U(y_k)]_\delta = \begin{pmatrix} b_{1,k} \\ b_{2,k} \\ \vdots \\ b_{r,k} \end{pmatrix}.$$

Definimos

$$[U]_\gamma^\delta = ([U(y_1)]_\delta \ [U(y_2)]_\delta \ \dots \ [U(y_n)]_\delta)$$

$$= \begin{pmatrix} b_{1,1} & b_{1,2} & \dots & b_{1,n} \\ b_{2,1} & b_{2,2} & \dots & b_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{r,1} & b_{r,2} & \dots & b_{r,n} \end{pmatrix}.$$

Por otro lado, tenemos que para cada $j = 1, \dots m$,

$$\begin{aligned} U \circ T(x_j) &= U(T(x_j)) \\ &= U(a_{1,j}y_1 + a_{2,j}y_2 + \dots + a_{n,j}y_n) \\ &= a_{1,j}U(y_1) + a_{2,j}U(y_2) + \dots + a_{n,j}U(y_n) \\ &= a_{1,j}(b_{1,1}z_1 + b_{2,1}z_2 + \dots + b_{r,1}z_r) \\ &\quad + a_{2,j}(b_{1,2}z_1 + b_{2,2}z_2 + \dots + b_{r,2}z_r) \\ &\quad + \dots + \\ &\quad + a_{n,j}(b_{1,n}z_1 + b_{2,n}z_2 + \dots + b_{r,n}z_r) \\ &= (a_{1,j}b_{1,1} + a_{2,j}b_{1,2} + \dots + a_{n,j}b_{1,n})z_1 \\ &\quad + (a_{1,j}b_{2,1} + a_{2,j}b_{2,2} + \dots + a_{n,j}b_{2,n})z_2 \\ &\quad + \dots + \\ &\quad + (a_{1,j}b_{r,1} + a_{2,j}b_{r,2} + \dots + a_{n,j}b_{r,n})z_r \end{aligned}$$

de donde

$$[U \circ T(x_j)]_\delta = \begin{pmatrix} a_{1,j}b_{1,1} + a_{2,j}b_{1,2} + \dots + a_{n,j}b_{1,n} \\ a_{1,j}b_{2,1} + a_{2,j}b_{2,2} + \dots + a_{n,j}b_{2,n} \\ \vdots \\ a_{1,j}b_{r,1} + a_{2,j}b_{r,2} + \dots + a_{n,j}b_{r,n} \end{pmatrix}$$

osea que

$$[U \circ T]_\beta^\delta =$$

$$\begin{pmatrix} a_{1,1}b_{1,1} + a_{2,1}b_{1,2} + \dots + a_{n,1}b_{1,n} & \dots & a_{1,m}b_{1,1} + a_{2,m}b_{1,2} + \dots + a_{n,m}b_{1,n} \\ a_{1,1}b_{2,1} + a_{2,1}b_{2,2} + \dots + a_{n,1}b_{2,n} & \dots & a_{1,m}b_{2,1} + a_{2,m}b_{2,2} + \dots + a_{n,m}b_{2,n} \\ \vdots & \ddots & \vdots \\ a_{1,1}b_{r,1} + a_{2,1}b_{r,2} + \dots + a_{n,1}b_{r,n} & \dots & a_{1,m}b_{r,1} + a_{2,m}b_{r,2} + \dots + a_{n,m}b_{r,n} \end{pmatrix}.$$

Por otro lado,

$$[U]_\gamma^\delta [T]_\beta^\gamma = \begin{pmatrix} b_{1,1} & b_{1,2} & \dots & b_{1,n} \\ b_{2,1} & b_{2,2} & \dots & b_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{r,1} & b_{r,2} & \dots & b_{r,n} \end{pmatrix} \begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,m} \\ a_{2,1} & a_{2,2} & \dots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \dots & a_{n,m} \end{pmatrix}$$

$$= \begin{pmatrix} a_{1,1}b_{1,1} + a_{2,1}b_{1,2} + \ldots + a_{n,1}b_{1,n} & \dots & a_{1,m}b_{1,1} + a_{2,m}b_{1,2} + \ldots + a_{n,m}b_{1,n} \\ a_{1,1}b_{2,1} + a_{2,1}b_{2,2} + \ldots + a_{n,1}b_{2,n} & \dots & a_{1,m}b_{2,1} + a_{2,m}b_{2,2} + \ldots + a_{n,m}b_{2,n} \\ \vdots & \ddots & \vdots \\ a_{1,1}b_{r,1} + a_{2,1}b_{r,2} + \ldots + a_{n,1}b_{r,n} & \dots & a_{1,m}b_{r,1} + a_{2,m}b_{r,2} + \ldots + a_{n,m}b_{r,n} \end{pmatrix}.$$

Concluimos que $[U \circ T]_\beta^\delta = [U]_\gamma^\delta [T]_\beta^\gamma$.