On distributions determined by their upward, space-time Wiener-Hopf factor

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The characteristic function φ of any probability distribution μ on $\mathbb{R}$ can be decomposed as

$$1 - s\varphi(t) = [1 - \kappa_+(s,t)] \cdot [1 - \kappa_-(s,t)]\,, \quad s \in [0,1)\,,\ t \in \mathbb{R}\,,$$

where κ_+ and κ_- are respectively the upward and downward space-time Wiener-Hopf factors of μ. The latter factors are defined by

$$\kappa_+(s,t) = \mathbb{E}(s^{T_+} e^{itS_{T_+}}) \ \text{ and } \ \kappa_-(s,t) = \mathbb{E}(s^{T_-} e^{itS_{T_-}})\,,$$

where (S_n) is a random walk with step distribution μ, starting at 0 and T_+, T_- are the first passage times above and bellow 0 by (S_n), that is $T_+ = \inf\{n \geq 1 : S_n > 0\}$ and $T_- = \inf\{n \geq 1 : S_n \leq 0\}$.

We prove that μ can be characterized by the sole data of the upward factor $\kappa_+(s,t)$, $s \in [0,1)$, $t \in \mathbb{R}$ in many cases including the case where 1) μ has some positive exponential moments, 2) the function $t \mapsto \mu(t,\infty)$ is completely monotone on $\mathbb{R}_+$, 3) the density of μ in $\mathbb{R}_+$ satisfies some conditions of analycity... We conjecture that any probability distribution is characterized by its upward factor. This conjecture is equivalent to the following: *Any probability measure μ on $\mathbb{R}$ whose support is not included in $\mathbb{R}_-$ is determined by its convolution iterations μ^{*n}, $n \geq 1$ restricted to $\mathbb{R}_+$.* In many instances, the sole knowlege of μ and μ^{*2} restricted to $\mathbb{R}_+$ is actually sufficient to determine μ.

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